

PHENOMENOLOGY AND THE FORMAL SCIENCES

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SÍNTESE – Este artigo procura mostrar que as idéias filosóficas de Husserl não apenas influenciaram o trabalho de alguns dos maiores matemáticos do século XX, mas foram decisivas para aproximarem uma epistemologia das ciências formais de uma fenomenologia do significado.

PALAVRAS-CHAVE – Ciências formais. Epistemologia. Fenomenologia. Matemática.

ABSTRACT – The article seeks to show that Husserl's philosophical ideas not only influenced the work of some of the greatest mathematicians of the 20th century, but were also decisive in bringing together an epistemology of the formal sciences and a phenomenology of meaning.

KEY WORDS – Epistemology. Formal sciences. Mathematics. Phenomenology.

I will consider here three aspects of the relation between phenomenology and the formal sciences (mathematics and logic): the relevance of philosophical problems in the foundations of mathematics for the development of Husserlian phenomenology, in particular in the period from the *Philosophie der Arithmetik* (PA, 1891) to the *Logical Investigations* (1900-01); the contributions of Husserl to the philosophy of formal sciences; and the usefulness of phenomenology for the mathematical practice.

According to Michael Dummett, "The philosophy of mathematics has, indeed, to be regarded as a specialized branch of philosophy. That it often has much to teach the rest of philosophy is due to the fact that problems arise in it which are analogues of problems that arise in other fields, and often in a form which makes the issues stand out sharper; the problems remains exceedingly difficult to resolve, and yet are more tractable than their analogues in other fields, just because attention is restrict to an area within which certain phenomena do not occur; we are, as it were, studying a part of the galaxy not obscured by dust clouds."¹

This is particularly true with respect to Husserl's philosophy. Mathematics was Husserl's first philosophical concern, and remained a strong focus of interest throughout his philosophical development. Many issues that are central to

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¹ Dummett, 1981, p. 666.

Husserl's philosophy surfaced first in connection with the philosophy of mathematics, and can be better understood from this perspective. One example is Husserl's rejection of psychologism in the *Logical Investigations*. As I will try to argue briefly below, this can be explained, at least in part, by the flagrant incapacity of psychologism to provide a convenient framework for the philosophy of arithmetic. Another example is the problem regarding symbolic reasoning in general. The need to account for symbolic knowledge in mathematics was certainly a major force in shaping Husserl's epistemology: purely symbolic mathematical reasoning raises the issue of purely intentional positing acts, which claim for a general theory of intentional acts.

In fact, as Husserl himself acknowledged in the preface to the first edition of the *Logical Investigations*, the philosophy of mathematics provided the motive of which this masterwork is the development. The attempt to fulfill the program of a psychologistic foundation for mathematics had led Husserl into a dead end. This program involved tying arithmetical concepts to certain mental operations (collecting and abstracting, in particular). It was carried out in part in his first major work (*PA*), but could not be extended, as Husserl realized when he tried, to more advanced areas of arithmetic. Although Husserl never claimed that numbers are *only* mental constructs, he cherished the idea of connecting numbers to the life of the mind by giving them the sort of reductive treatment he pursued in *PA*. When he realized that there are numbers, such as negative, irrational and complex numbers (which he called *imaginary*), that cannot be given a genesis in terms of mental operations, it became too obvious that there was a gap between the objectivity of mathematical concepts and the mental life of a subject that he did not know how to fill.

The thematic focus of *PA* was the concept of number, non-negative integers to be more precise. At that time numbers were for Husserl simply the quantitative forms of quantitatively determined collections of objects, regardless of their nature; that is, answers to the question "how many". For Husserl, these forms are residues of an active process taking place in *real* consciousness, which involves collecting objects and abstracting from their nature and order, more or less in the spirit of Cantor's set theory. The never published second volume of *PA* should have dealt with imaginary numbers, but at this point the entire project came to a halt. Imaginary numbers simply could not be treated like the non-negative integers. The symbolic nature of mathematical knowledge, which can easily give rights of citizenship to entities that cannot be given a genesis in consciousness, like imaginary numbers, stood in the way of a successful psychologistic foundation of arithmetic. The ties between the objective world of mathematics and the life of consciousness that were central to the project of *PA* were severed, and a major philosophical problem remained: to root objectiveness in subjective consciousness.

Imaginary numbers also disclosed to Husserl the essential nature of mathematical knowledge. He realized that mathematics is not only essentially *symbolic* but, more importantly, *formal*. How can imaginary numbers, Husserl asked, which do not have, and cannot have a proper denotation contribute nonetheless to our

knowledge of numbers proper? Or, more dramatically, how purely symbolic mathematics, which is apparently a mere game with symbols abiding only to the law of non-contradiction, can contribute with any kind of knowledge at all? Although, Husserl answers, imaginary symbols play only a secondary role in mathematics, which we can very well do without, that of making calculations easier, purely symbolic mathematics gives us something much more important, namely, *formal knowledge*.

So, Husserl concluded, formal mathematics is a sort of a *priori* knowledge of the formal properties of domains of objects. In other words, a sort of logic, not of propositions but of objective forms. Logic and mathematics are then closely related disciplines, both are *a priori* and analytic², both are formal and objectively valid. After the failure of the project of *PA*, Husserl realized he had a new task: to start from scratch and submit logic in general to a fresh philosophical scrutiny that took into consideration its *objective* nature, hence a task to be performed against psychologism. But subjectivity could not be ignored simply, it had to appear in the picture somehow if the applications of logic and mathematics were to be accounted for. The task was immense, and the *Logical Investigations* are Husserl's report of how he carried it out.

Given the role foundational mathematical problems played in the genesis of the *Logical Investigations* (which I am here dramatizing a bit for the sake of making a point very clearly), it is not surprising that we can find in this *opus magnum* a very well developed philosophy of mathematics. The nature of mathematical knowledge and mathematical objects, the insertion of formal mathematics into logic as *formal ontology*, the nature of mathematical intuition and the semantics of mathematical statements, to name only a few, are among the many issues in the philosophy of mathematics that Husserl considered and discussed in detail. It is worth noticing that Husserl's attempts to solve the problem of imaginary numbers gave him the concept of a formal multiplicity, as well as the notions of syntactic completeness and a form of semantic completeness (the non extensibility of the domain of a formal deductive system by the adjunction of new elements) that were central to the new formal mathematics and metamathematics that were being created at the time by Hilbert. Hilbert himself had also put forward similar views but Husserl seems to have arrived at them independently. Although we only know Husserl's complete solution for the problem of imaginary numbers from the sketches for a series of two talks he gave, invited by Hilbert, to the Mathematical Society in Göttingen in 1901, this solution was already in his possession since at least 1891, as can be inferred from his review of Schröder's *Lessons on the Algebra of Logic*, and a letter to Frege, both from that year. The fact is that after moving to Göttingen in 1901 and entering the circle of Hilbert, Husserl certainly exerted some influence on, but was also influenced by this great mathematician. Ho-

² In a Bolzanian rather than Kantian sense; for Husserl an analytic judgement is one that is true in virtue of its form only.

wever, the extend of this mutual influence is not easy to assess and remains an open problem for historians of philosophy and mathematics.³

Husserl's interest in the philosophy of mathematics was no accident. He studied astronomy in Leipzig from 1876 to 1878, mathematics in Berlin from 1878 to 1881 with Kronecker and Weierstrass, of whom he was for a period an assistant, and also in Vienna, where he obtained a doctorate with a thesis on the Calculus of Variations in 1883.

Husserl was for many years in close contact with mathematical giants of the stature of Hilbert, Frege and Cantor, who belonged to the committee of his thesis of *venia legendi* on the concept of number in 1887. His mathematical knowledge was that of a professional mathematician. He was able to appreciate the important developments in mathematical logic that were being carried out at that time by, among others, Boole and Frege, as well as the creation of metamathematics by Hilbert and the invention of set theory by Cantor, when for the first time the notion of an actual infinity was coherently developed. All these theories posed many difficult philosophical problems that claimed for a solution. Husserl set out to tackle some of them in his *Habilitationsschrift* of 1887 and *Philosophie der Arithmetik*, and this was the beginning of his philosophical career.

In the *Logical Investigations* (whose central ideas were already in place by 1895-6) we can find the views on the nature of logic and mathematics that Husserl endorsed to the end of his life. Logic, which according to him includes the whole of formal mathematics, is understood as a *mathesis universalis* in the sense of Leibniz. This *mathesis* is divided in two parts, corresponding respectively to the notion of meaning (*apophansis*) and object (*formal ontology*, which includes formal mathematics and some contentual mathematical theories, like set theory and arithmetic). Each part is structured in three levels: the level of morphology, the level of theories of morphological concepts and the level of the meta-theory of these theories, the *Mannigfaltigkeitslehre*. The task of morphology, in the case of *apophansis*, is to fix the meaning categories, such as the concepts of name, concept and proposition, and, in the case of formal ontology, the ontological categories, such as the concepts of number and class, and disclose the *a priori* laws of their syntax.

The mathematical theories of the concepts of number (arithmetic) and class (set theory), as well as mereology (the theory of wholes and parts) or the theory of relations are placed in the second level of formal ontology, for morphological categories are their subject matter. Purely formal mathematics (abstract algebra) and the meta-theory of formal domains of formal deductive systems are placed in the third level of formal ontology. Mechanics and physical geometry (but not formal geometry) fall outside the *mathesis*, for they are only *regional* ontologies (the re-

³ For details, see da Silva, 2000b.

gion of forces, in the case of mechanics, or spatial forms, in the case of geometry). It is interesting to notice that the idea of a *Mannigfaltigkeitslehre* was strongly influenced, as Husserl himself acknowledged, by mathematicians such as Cantor, Riemann, Lie, Grassmann, Klein and Helmholtz, which is another aspect of the influence of mathematics in the shaping of Husserl's thought.⁴

Let us concentrate now on Husserl's contributions to the epistemology of mathematics. One of the most important is, of course, his account of mathematical intuition, which is understood simply as the way to gain access to mathematical objects, or still the mathematical correlate of sense perception. We can put in a simple formula what Husserl takes it to be: mathematical intuition = categorial intuition (categorial perception or categorial imagination) + categorial abstraction. Let us provide an example, we can see that John is taller than Peter, the seeing of this relation beyond the seeing of John and Peter only is an instance of categorial perception. We can now abstract the matter of this categorial percept and retain only its form: x is taller than y . Or go even further and abstract the nature of the relation altogether, keeping only the idea of a relation: x is in a relation R to y . This is an instance of categorial abstraction (or still, formal abstraction), it gives us the categorial forms mathematics is about. These forms are nothing but dependent moments of certain categorial objectualities, namely states-of-affairs. Having now the form of a binary relation, which can be generalized to a form of relation admitting any number of entries, mathematics will say all that can be said about it, considering only that it is the form of a relation. It will study all their possible formal properties – such as reflexivity, symmetry, connectness, transitivity, etc. – as well as all the materially indeterminate objective domains in-formed by these relational forms. In short, it will provide a theory of relations.

Mathematical knowledge is, according to Husserl, not only *a priori*, but also analytic, in the sense of the *Third Investigation*: that which is true and remains true under formal abstraction. For example, $7 \text{ apples} + 5 \text{ apples} = 12 \text{ apples}$ is an analytic judgement because $7x's + 5x's = 12x's$, no matter what x denotes. The mathematical truth that $7 + 5 = 12$ is analytic for the same reason, not for there being a relation of inclusion between the subject and the predicate of the judgement, which in this case are not even clearly defined. Mathematical truths are analytic (against Kant) for they are true by virtue of their form alone. The assertion $7 + 5 = 12$ involves only numerical forms and gives us knowledge concerning these forms, it provides us with a type of formal knowledge we can apply to no matter which collection of objects we may be interested in. This is why arithmetic, not only purely formal arithmetic, but also contentual arithmetic, that is, the arithmetic of the usual numbers, is for Husserl a formal ontological discipline.

It is clear from this example that mathematical knowledge of the type provided by contentual mathematical disciplines relies on mathematical intuition as a

⁴ For details, see da Silva, 1999, 2000a and Ortiz Hill, 1995, 2000.

provider of the forms these theories are about. But we may also create mathematical forms by pure stipulation, as purely formal mathematics does. Husserl was one of the first philosopher to realize that the freedom to create new mathematical forms was at the core of Hilbert's formalist approach to mathematics, something Frege never quite realized. Of course, even contentual mathematical theories, like arithmetic, cannot depend wholly on intuition. Huge numbers, for instance, cannot be given appropriately in intuition. But they can at least be partially intuited, and moreover there is some connection between the symbolic and intuitive aspects of arithmetic (namely, formal similarity) that lack in purely symbolic theories.⁵

Let us say now a few words about Husserl's contribution to the semantics of mathematical statements. The most relevant is the distinction between state-of-things (*Sacherverhalt*) and situation-of-things (*Sachlage*). States-of-things are the reference of statements, situation-of-things are pre-categorical objectualities that underlie states-of-things, in such a way that the same situation-of-things corresponds to possibly many logically equivalent states-of-things. For instance, the assertions that $2 < 3$ and $3 > 2$ refer to two different states-of-things, but the same situation-of-things underlies both of them. This situation, however, cannot be appropriately described, for descriptions are statements, and statements refer only to states-of-things. Situations-of-things are somehow ineffable. Husserl envisaged the possibility of using this semantic distinction to treat the phenomenon of equivalent, although different physical theories. There is however a similar phenomenon in mathematics, which has gone so far almost unexplored and unaccounted for, the interderivability of distinct and sometimes apparently unrelated mathematical assertions, like the axiom of choice and Tychonoff's theorem, for instance. Obviously, there is some single situation these two mathematical assertions describe, as from two different perspectives. Husserl's semantic distinction offers the philosopher a convenient way of dealing with this question.⁶

But Husserl's contribution to the philosophy of mathematics does not stop here. He also presented in an appendix to *The Crisis of European Science and Transcendental Phenomenology* (1936) entitled *The Origin of Geometry*, his contribution to the theory of the transcendental genesis of mathematical theories. He showed there how objective mathematical theories of ideal objectualities have their origins in transcendental consciousness, having at their bases a world of things and practices (the *Lebenswelt*), and gain ideal status in the intersubjective world of culture and language. Something similar is presented with respect to logic in *Experience and Judgement* (1939) and, to a certain extent, also in *Formal and Transcendental Logic* (1929). In a way this is the extension of Husserl's account of mathematical intuition to transcendental subjectivity understood now in

⁵ With respect Husserl's epistemology of mathematics see da Silva 1993 and Rosado Haddock, 1987.

⁶ For details see Rosado Haddock, 1991 and 1992.

terms of intersubjectivity. It is also a case study of the relation between objectivity and subjectivity that challenged him at the beginning of his philosophical career. That Husserl gave such a detailed attention to the mathematical version of this problem is probably a tribute to the fact that it first appeared to him in this context.

Much more can and has been said about the many aspects of Husserl's contribution to the philosophy of formal sciences and the role they have played in the development of phenomenology. My goal in these few pages was to give a short account of some of them and direct the reader to the extensive bibliography on the subject, of which some items are listed here. But before closing let me say something about the influence of Husserl's phenomenology on mathematics, not only on its philosophy, but on its practice as well.

Mathematics is an area in which philosophy has always played a major role, which nonetheless has frequently been overlooked by shortsighted commentators. But it only takes a *vol d'oiseau* over the history of philosophy for one to see that mathematics and philosophy have always been engaged in a wonderfully productive dialogue. The philosophies of Leibniz and Descartes, who are counted among the greatest philosophers as well as the greatest mathematicians of all times, can hardly be well understood if one is unable to appreciate their mathematical creations, and conversely, the appreciation of their mathematics is enhanced by the knowledge of their philosophical ideas. Kant's philosophical reform was launched, as we all know, by the problem concerning the possibility of mathematical knowledge, and conversely, Kant's epistemology played an important role in the development of constructive mathematics. The systems of Plato, Bolzano or Spinoza also own so much to mathematics that it is difficult to think of them independently of this aspect (Plato himself warned us against trying to gain access to his philosophy without knowing geometry, and he meant it.) Conversely, the invention of the infinitesimal calculus by Leibniz, or the contributions of Descartes to geometry make much more sense in the context of their philosophical ideas. Closer to us, the intuitionist mathematics of Brouwer cannot be fully appreciated cut off from his epistemology and Frege's philosophy isolated from his work in the foundations of mathematics, or vice-versa, will inevitably lack something. The examples are just too many.

Something of the sort holds true for Husserl. If one chooses, either out of ignorance or prejudice (or worse, both) to overlook the importance mathematics played in the development of Husserl's philosophy, then one is just choosing to ignore one important element for understanding this development (and self-inflicted blindness is a terrible thing).

Husserl's philosophical ideas also influenced the work of some of the greatest mathematicians of the 20th century. I have already mentioned above the not yet completely understood mutual influence between Husserl and Hilbert. But this is not the whole history. Hermann Weyl and Kurt Gödel, mathematicians who were not surpassed last century by any other in terms of importance, explicitly ac-

knowledged the impact Husserl's ideas had on them and their work. Weyl followed some of Husserl's lessons on the philosophy of time, and this helped him to shape his approach to analysis as a theory of the linear continuum, of which intuitive time is the best example.⁷ Gödel, who read Husserl extensively from 1959 on, extracted from him two notions basically, that of a categorial form of intuition and the idea of philosophy as a rigorous science. Both could easily be accommodated, Gödel believed, into his own epistemology of mathematics, which went hand in hand with his mathematical creation.

Gödel in fact applied Husserl's ideas more to his philosophizing than to his mathematical work proper. Weyl, on the other hand, actually carried out a phenomenological analysis of time as a prolegomenon to his reconstruction of the mathematical theory of the continuum. This stands out, I believe, as a model for the application of phenomenology in mathematics. I think that phenomenological analysis can be put to the service of mathematics insofar as it brings to light the intentional structures that "frame" our mathematical concepts. By so doing it helps to reveal the pre-theoretical notions that underlie the thematic concepts of our mathematical theories, which are clarified and possibly improved in the process. In short, phenomenology can serve as a *prima philosophia*, or a founding enterprise, also in the domain of formal sciences.

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⁷ See Weyl, 1987 and da Silva, 1993.

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