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How structuralism can solve the 'access' problem*

Como o estruturalismo pode resolver o problema do 'acesso'

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Abstract: According to mathematical structuralism, the subject matter of mathematics is not the study of mathematical objects, but of mathematical structures. By moving away from objects, the structuralist claims to be in a position to solve the 'access' problem: structuralism explains the possibility of mathematical knowledge without requiring any access to mathematical objects. Fraser MacBride has challenged the structuralist response, and argued that the structuralist faces a dilemma in the attempt to solve that problem (MacBride, 2004). In the present paper, I argue that MacBride's dilemma can be resisted, and that, particularly in the version articulated by Michael Resnik (RESNIK, 1997), structuralism can solve the 'access' problem. I show exactly how MacBride's dilemma fails, and argue that this failure provides an opportunity to highlight a significant feature of structuralism: the way in which it articulates a fundamentally different picture of mathematical epistemology than traditional epistemology would suggest.

Keywords: Mathematical Epistemology. Ontology. Platonism. Structuralism.

Resumo: De acordo com o estruturalismo matemático, a matemática não consiste no estudo de objetos matemáticos, mas de estruturas. Ao afastar-se dos objetos, o estruturalista reivindica uma posição que lhe permite resolver o problema do "acesso": é possível explicar a possibilidade do conhecimento matemático sem exigir qualquer acesso aos objetos em questão. Fraser MacBride criticou a resposta

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estruturalista, argumentando que esta enfrenta um dilema na tentativa de resolver o problema em apreço (MACBRIDE, 2004). Neste artigo, argumento que o dilema de MacBride pode ser resistido e que, especialmente na versão articulada por Michael Resnik (RESNIK, 1997), o estruturalismo pode resolver o problema do "acesso". Mostro exatamente como o dilema de MacBride falha, argumentando que esta falha nos fornece uma oportunidade para destacar uma característica importante do estruturalismo, a saber: a maneira pela qual ele articula uma imagem fundamentalmente diferente da epistemologia matemática com relação àquela sugerida pela epistemologia tradicional.

Palavras-chave: Epistemologia Matemática. Estruturalismo. Ontologia. Platonismo.

1 Introduction

A major problem has driven much of recent (and also not so recent) philosophy of mathematics: the 'access' problem. How can we explain mathematical knowledge, or even the reliability of mathematical beliefs, given that we have no access to mathematical entities (see BENACERRAF, 1973 and FIELD, 1989)? Traditional epistemological accounts rely on the otherwise reasonable assumption that the objects we study should play a role in the way in which we come to know these objects. But without any sort of access to mathematical objects, it seems that the epistemology of mathematics has to provide a very different account. Structuralists in the philosophy of mathematics, such as Michael Resnik (see his, 1997), who insist that the subject matter of mathematics is structure (or patterns) rather than objects, argue that they are in a good position to solve the access problem. After all, on the structuralist picture, no access to mathematical objects is required for the articulation of a mathematical epistemology – a different strategy is devised in which patterns are central, instead of mathematical entities.

Fraser MacBride has challenged the structuralist response, and argued that the structuralist faces a dilemma in the attempt to solve the 'access' problem (MACBRIDE, 2004). In the present paper, I argue that MacBride's dilemma can be resisted. I provide a framework in terms of which it becomes clear how structuralism can solve the 'access' problem – particularly in the version articulated by Resnik. It's instructive to see, however, exactly how MacBride's dilemma fails, since this provides an opportunity to highlight a significant feature of structuralism: it provides a fundamentally different picture of mathematical epistemology than the one found in traditional epistemological accounts.

2 An epistemological challenge: the 'access' problem

The crucial feature of mathematical structuralism is to conceptualize mathematics as the study of structures, rather than objects. Different forms of structuralism provide different accounts of structure (see, e.g., RESNIK, 1997 and SHAPIRO, 1997). Roughly speaking, a structure can be thought of as a certain domain (that need not be a set) and a family of relations formulated on such domain. But, crucially for the structuralist, it doesn't matter which mathematical objects one considers, as long as they satisfy the relevant structure, that's sufficient to explain the possibility of mathematical knowledge.

We find this move in Resnik's defense of structuralism. To explain the possibility of mathematical knowledge, Resnik introduces the notion of a template, which is a concrete entity – that includes drawings, physical models, blueprints – and is meant to link the concrete aspects of our experience with abstract patterns (Resnik's term for structure). The crucial idea is that there are structural relations (such as isomorphisms) between templates and patterns that allow us to represent the latter via the former. In particular, it's because there are such structural relations between patterns and templates that mathematicians can use proofs – the process of creating and manipulating concrete templates via certain operations – to generate information about abstract patterns (RESNIK, 1997, p. 229-235). And given that mathematicians only have access to templates, no direct access to positions in patterns – that is, no direct access to mathematical objects – is presupposed in Resnik's picture.

A significant feature of patterns, on Resnik's view, is the fact that the positions in such patterns are incomplete. This means that there is no fact of the matter as to whether these positions have certain properties or not. Consider, for example, the second position in the natural number pattern (for simplicity, call that position "the number 2"). It's not clear that there is a fact of the matter as to whether this position – the number 2 – is the same as the corresponding position in the real number pattern. In other words, it's not clear that there is a fact of the matter as to whether the number 2 in the natural number pattern is the same as the number 2 in the real number structure. After all, the properties that a position in a patter has depend on the pattern to which it belongs. In the natural number pattern, the number 2 has the third position in the pattern – that is, the number 3 – as its immediate successor. But this isn't the case in the context of the real number pattern. Of course, in the real number pattern, the immediate successor of the number 2 *that is also a natural number* is the number 3. But to say that this is the same property as the one in the natural number pattern is already to assume that the corresponding numbers are the same – which is the point in question. As a result, it's not clear how one could decide issues such as these[1].

Given the significance that Resnik assigns to patterns as vehicle of mathematical information, it would be a rather unexpected result if, in the development of an epistemological account of mathematics, patterns fail to play any epistemological role. But this is ultimately the thrust of MacBride's dilemma (2004, p. 315-316).

According to MacBride, either templates and patterns can be structurally related to each other, or they cannot. (A) If they can be structurally related, then templates and patterns do have an information-bearing connection (and so, the former can be invoked in developing an epistemology for the latter). However, (B) it then also follows that the positions under consideration are not incomplete (as opposed to what Resnik claims). After all, (C) "it is a precondition of positions enjoying the structural relations in question that they belong to the very same universe of discourse as the ordinary concrete objects from which templates are made" (MACBRIDE, 2004, p. 316).

Alternatively, (D) if templates and patterns cannot be structurally related, then it's unclear how there could be an information-bearing connection between them. After all, (E) Resnik doesn't reveal which connection is that, and it's inexplicable how "objects that do not *even* belong to the same universe of discourse could bear such a[n information-bearing] connection to one another" (MACBRIDE, 2004, p. 316). In either case, there's trouble.

MacBride's concludes that Resnik, despite his protestations to the contrary, ended up demanding some sort of interaction between the mathematicians and the objects of mathematical investigation. This, in turn, led

> Resnik to seek a worldly embodiment of the patterns – the templates – that we can literally confront. But the patterns cannot be embodied (on pain of being complete), and Resnik is left with an impossible combination of views (MACBRIDE 2004, p. 316).

Is there a way out?

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1 Note that there are two senses of incompleteness of a position in a pattern: (i) There is an *ontological* sense, according to which there is no fact of the matter as to whether that position has certain properties or not. But (ii) there is also an *epistemic* sense, according to which we do not have complete information to decide whether a given position has or not certain properties. Clearly, (i) entails (ii), but not vice-versa; (i) is thus more fundamental.

3 Solving the 'access' problem

I think there is a way of overcoming the difficulty. Properly characterized in terms of the framework presented below, mathematical structuralism has the resources to resist MacBride's dilemma. First, note that statement (B) – the claim that positions in a pattern aren't incomplete – doesn't follow from premise (A) – the claim that if patterns and templates are structurally related, then there is some information-bearing connection between them. To see why this is the case, recall that there are several forms of structural relations between patterns (or between templates and patterns): they include not only isomorphism and homomorphism, but also *partial* isomorphism and *partial* homomorphism. And there are formal characterizations of the latter partial structural relations in which it's simply *not* the case that the objects related by the appropriate mappings have to be complete (see BUENO, FRENCH and LADYMAN, 2002; and DA COSTA and FRENCH, 2003; see also FRENCH, 2014).

Consider, for example, a partial structure $\langle D, R_i \rangle_{i \in I}$, where D is a non-empty set and R_i, $i \in I$, is a family of partial relations. A partial relation R_i, $i \in I$, over D is a relation which is not necessarily defined for all n-tuples of elements of D. The partiality of these relations can be interpreted in two ways: (i) It can be interpreted *ontologically*, as representing the incompleteness or partialness of the relations linking the elements of D. Or (ii) the partiality can be interpreted *epistemically*, as representing the incompleteness or partialness of our information about the actual relations linking the elements of D. (The formalism suggested here is neutral on this issue, and as will become clear, it can be interpreted in either way.) More formally, each partial relation R can be viewed as an ordered triple $\langle R_1, R_2, R_3 \rangle$, where R_1, R_2, and R_3 are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^n$, and such that: R_1 is the set of n-tuples that (we know) belong to R; R_2 is the set of n-tuples that (we know) do not belong to R; and R_3 is the set of n-tuples for which it is not defined whether they belong or not to R. (Note that when R_3 is empty, R is a normal n-place relation that can be identified with R_1.)

A notion of quasi-truth for partial structures can also be defined – this notion will be important for the discussion below. Given a partial structure A, there are several total structures B that extend the partial relations in A to total relations (relations that are defined for all n-tuples of elements of D). These are called *A-normal structures*. We then say that a sentence α is quasi-true in a partial structure A if there is an A-normal structure B in which α is true (in the Tarskian sense).

In terms of partial structures, it's possible to define various forms of partial morphisms (such as partial isomorphism and partial homomorphism) that extend very naturally the usual notions of isomorphism and homomorphism to partial contexts. Let $S = \langle D, R_i \rangle_{i \in I}$ and $S' = \langle D', R'_i \rangle_{i \in I}$ be partial structures, where R_i and R'_i are (for simplicity) binary partial relations. We say that a partial function $f: D \to D'$ is a *partial isomorphism* between S and S' if (i) f is bijective, and (ii) for every x and $y \in D$, $R_1xy \leftrightarrow R'_1f(x)f(y)$ and $R_2xy \leftrightarrow R'_2f(x)f(y)$. So, when R_3 and R'_3 are empty (that is, when we are considering total structures), we have the standard notion of isomorphism. Moreover, we say that a partial function $f: D \to D'$ is a *partial homomorphism* from S to S' if for every x and every y in D, $R_1xy \to R'_1f(x)f(y)$ and $R_2xy \to R'_2f(x)f(y)$. Again, if R_3 and R'_3 are empty, we obtain the standard notion of homomorphism as a particular case.

To illustrate how this framework is used, suppose, for a moment, that I'm engaged in the business of weather simulation, and that I use a template to describe the weather. The template's components are descriptions of particular states of the weather in a given location at a certain moment in time. Note that the template I use is a *concrete* object: both the template itself (a string of symbols in my computer) and the objects it describes (the temperature, pressure, and humidity of the location under study) are concrete[2]. More importantly, note also that my template is *not* complete (it can be easily represented by a given partial structure). It doesn't include *all* the information about the weather, even in the location I'm studying. The template includes *some* information regarding the weather – the one selected for depiction (represented in terms of R_1- and R_2-components). But not all aspects of the weather are considered: say, factors leading to lightning may be ignored. And even those aspects that *are* considered are not complete: water temperatures in the central-equatorial Pacific Ocean might not be available (they are elements in the R_3-components).

After entering the data regarding a particular configuration of the weather, I run my simulation. At this point, I *embed* my template into an abstract pattern that includes various equations that describe the weather evolution. So, there are structural relations between my template and the abstract pattern: I map, via a partial isomorphism, or a partial homomorphism, certain data in the template into the abstract pattern. After running the program, I get some results, and interpret them back into the world. The results describe one way in which the weather may evolve. Does it follow that the positions in question are complete? Clearly not. The positions weren't complete in the template to begin with,

2 Of course, I use numbers (actually, numerals) to represent the particular state of the weather in a given moment. But the temperature, pressure, and humidity, although represented by numbers, are *not* themselves numbers. They are *physical* states in the world.

and they aren't complete when I get my results back. So, despite the information-bearing connection between the templates and the patterns, it doesn't follow that the positions in questions were not incomplete.

Note that an isomorphism (or, *mutatis mutandis*, a partial isomorphism) is an information-bearing connection between patterns, or between templates and patterns, in the sense that any two isomorphic structures are elementarily equivalent – that is, the same sentences are true in both structures. A similar point also holds for partial isomorphism: any two partially isomorphic structures are quasi-elementarily equivalent – that is, the same sentences are quasi-true in both structures. This allows one to transfer information from one structure to the other. But an isomorphism, or a partial isomorphism, doesn't require any sort of *interaction* between the objects related. It's simply a mapping between patterns that preserves certain structure. Nothing more is added. This allows the structuralist to provide an information-bearing connection between templates and patterns without requiring interaction between the positions in question or demanding that the positions in question be complete.

Exactly the same process goes on in the case of a mathematical proof. By manipulating certain interpreted symbols via truth-preserving inference rules, a mathematician shows that if the principles that are assumed in the proof are true, so is the result that follows from them – namely, the theorem that is established using the principles and the inference rules in question. This is done without having to establish any *access* to the mathematical objects referred to in the theorem. After all, the mathematician *doesn't* have to *establish the truth* of the principles from which the theorem follows. To prove the theorem, all that is needed is to establish a (consequence) relation between the statement of the theorem and the principles in question via the inference rules.

As an additional example, consider the notion of set and of *Urelemente* (objects that are not sets), and the different ways in which set-theoretic notions can be extended. Even though there are structural relations between different set-theoretic patterns that include *Urelemente* – and hence there are information-bearing connections between them – this doesn't guarantee that the sets in question (as positions in a pattern) are complete. And they aren't, since they can always be extended further. This doesn't require that even the *Urelemente* are complete. For all we know, they may even be quantum particles that lack well-defined identity conditions. (Of course, such particles are still *concrete* objects, spread out in space and time.)[3]

3 Note that it's not only positions in patterns that are not complete, in the sense that several of its properties can only be determined once a given pattern is specified. Concrete objects

Now, in support of (B) – the claim that positions in a pattern aren't incomplete – MacBride invokes (C), that emphasizes that for positions in patterns to share structural relations with concrete objects in templates, these positions have to be at the same universe as the objects that constitute the templates. However, the fact that positions in patterns and the corresponding objects in a template share the same universe of discourse doesn't entail that such objects, or the positions in question, aren't incomplete[4]. The examples of the weather template and of set theory with *Urelemente* clearly illustrate that: we have templates and patterns (or, in the case of set theory, different patterns) that share the same universe of discourse, but whose corresponding objects are incomplete. So, the inference from (C) to (B) also fails. And, as a result, it's clear which horn of the dilemma the structuralist blocks.

Given the description above of how structuralists can approach the access problem, can we say that Resnik has presupposed that mathematicians need to *interact* with the objects of mathematical investigation (MacBride [2004], p. 316)? I don't think so. Mathematicians clearly interact with templates. But these are *concrete* objects, and there's no claim that templates, in turn, provide any means of interaction with mathematical objects. Not only talk of these objects drops out in the structuralist picture (the emphasis goes to patterns instead), but more importantly, it's possible to establish structural relations between templates and patterns without assuming any *interaction* with positions in a pattern, and so *no access* to these positions is presupposed. As noted, partial isomorphism and partial homomorphism provide just those structural relations.

4 Objections and responses

In support of the proposal just sketched, I'll consider a few objections and indicate my responses. This will offer an opportunity to highlight additional traits of the suggested view[5].

may also share the same feature, for familiar Quinean reasons: to determine whether a rabbit is present, and distinguish it from temporal stages of a rabbit, or undetached rabbit parts, one needs a background theory.

4 It doesn't entail that there is any interaction between these objects either. The universe of discourse only provides a collection of objects selected to be talked about. The collection may be as gerrymandered, disjoint, or disunified as it gets, with no interaction whatsoever among the objects.

5 My thanks go to anonymous reviewers and for commentators on earlier versions of this work for raising these points.

4.1 *The unavailability of morphisms*

It may be objected that there aren't isomorphisms or partial isomorphisms between templates, which are, as we saw, concrete objects, and patterns, which are abstract structures. After all, since the former are not structures themselves the notion of isomorphism, as defined above, cannot be applied to them.

In response, it's important to note that we have a *description* of the template that highlights the relevant relations among the entities that characterize that template. And the description defines a structure, with a given domain of objects (those selected for representation) and a family of relations (holding between the objects in question). And to such a notion of structure, one can readily apply the notions of isomorphism and partial isomorphism.

It may also be objected that it's unclear that there can be an isomorphism (or even a partial isomorphism) between a *finite* template and an *infinite* pattern, and thus the former are inadequate to capture the content of the latter.

In response, there can't indeed be such isomorphisms between structures of different cardinalities. That's why the notion of partial *homomorphism* was invoked in this context. With this notion in place, there's no difficulty to establish a suitable morphism between structures whose cardinality differs.

4.2 *Templates and existence*

One may worry about how to guarantee that by introducing a template – say, by characterizing certain comprehension principles – the structures that are thus introduced correspond to independently existing abstract patterns; that is, patterns that exist independently of any description provided by a template.

In response, I don't think we can have that guarantee, and that's why we shouldn't be Platonist about patterns. In fact, Resnik's overall epistemological strategy as well as his central structuralist insight (regarding the inherent incompleteness of mathematical objects) can all be preserved without a Platonist ontology. In other words, we can claim that mathematics is the study of patterns, and that the positions that constitute such patterns need not have identity conditions defined outside the context of such patterns (in this sense, such positions are incomplete). When mathematicians introduce comprehension principles, they define a context that provides meaning to the terms that are used in the relevant principles. *Within this context*, it's analytically true that there are objects and structures of the appropriate kind (introduced by the relevant comprehension principles).

However, this is not sufficient to guarantee the existence of objects and structures *independently* of the context in which such terms are introduced. We describe the practice as a matter of formulating such principles and exploring the connections between them. In exploring such connections, (partial) isomorphism and (partial) homomorphism are, of course, central. And so, we can have radically incomplete objects (radically incomplete positions in a pattern) without having a commitment to the existence of objects that are independent of the comprehension principles themselves. As a result, we can have structuralism without Platonism. It seems to me that that position is perfectly coherent, and can be motivated from mathematical practice in interesting ways (see BUENO, 2009).

Given these considerations, we need not be committed to the existence of patterns (independently of the framework determined by suitable comprehension principles). We never leave the context of templates that are determined, in the case of pure mathematics, by appropriate comprehension principles.

4.3 *Objects, 'places' in a structure, and incompleteness*

Given the lack of commitment to 'places' in a structure (the structuralist reconceptualization of objects), what sort of entities are such 'places'? And how can they be incomplete?

In response, on a structuralist view, the so-called 'places' in a structure just play a particular role, namely, the one that has been traditionally assigned to objects. But it's crucial that such 'places' never be reified. The whole point of a structuralist approach to mathematics is to indicate that 'places' are entirely dispensable and there's no need to settle any metaphysical question about their nature in order to make sense of mathematical practice. In particular, the question regarding what kinds of entities 'places' in a structure are becomes otiose, since no commitment to such 'places' is forthcoming in the resulting view.

As noted above, 'places' are incomplete given that not all of their properties are determined by the specification of the features that a template assigns to them. For instance, it's not determined by the characterization of the third position in the natural number structure that that position is the same as the third position in the fragment of the real number structure corresponding to the positive integers. The relevant structures simply fail to settle this issue. Usually, it is simply stipulated that these 'places' are the same. But that's simply a convenient choice, and no ontological conclusion can be derived from it.

4.4 *Templates, temperature, and concreteness*

It may be argued that templates are not concrete. Despite the fact that their domain may contain only concrete objects, the relations that, in a template, manage to represent structures are abstract; in fact, such relations are often highly idealized entities. How can they be concrete?

In response, it's important to note that several relations are clearly concrete. As I type these words at 30,000 feet above the ground, the relation my body bears to the ground (namely, *being above of*) is clearly concrete: it depends only on the relative spatial position between two concrete objects: my body and the ground. Even the relation *being 30,000 feet above of* is concrete, despite the use of the mathematical vocabulary to express it. I may need to invoke some mathematics to express and represent the exact distance between my body and the ground, but it is still a concrete fact that I bear to the ground, being positioned with respect to it in a particular cluster of relative locations.

A similar point can be made for templates used in mathematics. They are concrete objects (drawings, physical models, blueprints) that are used to represent relations among abstract structures (patterns). The relations among the items in a template are similarly concrete (think of the notation invoked in the drawing of a commutative diagram). Despite being concrete, these relations can still be, and typically are, used to represent corresponding relations in a pattern (an abstract structure) via the elementary equivalence (or partial elementary equivalence in the case of partial morphisms) of isomorphic (or partially isomorphic) structures.

I noted above that temperature and other such magnitudes are concrete. It may be objected that they cannot be. There are at least a couple of reasons for this claim. (a) Thought of as states of a physical system, all such magnitudes are abstract. On this conception, abstract objects are taken to be ontologically dependent entities, which in this instance depend on concrete things. (b) The concept of *temperature* is clearly idealized. It cannot be specified independently of idealizations that emerge from the limitations of what we can perceptually experience and what we cannot experience directly. We are sensitive to variations in the sensation of warmth and thermal equilibrium, but we cannot experience these differences directly in a precise scale. To transform these differences in differences of temperature that can be represented by real numbers, we need (i) to idealize our capacity of experiencing variations in the sensation of warmth, and (ii) assume that these variations are continuous. Temperature can then be thought of in terms of the class of all objects of the same temperature (that is, all objects that are in thermal equilibrium). At this point, suppose that a given body is in a determinate thermal state. One can then assign a number to that state, and more generally, assign numbers to thermal states of all kinds of objects (their "temperature"). These numbers thus represent a family of objects with regard to their thermal relation to another family of objects. As a result, it's unclear how temperature can be a concrete entity, since it cannot be formulated without indispensable reference to abstract objects.

In response, it's important to distinguish the concept of *temperature* from the physical process that this concept tries to measure. Understood as a physical process in the world, temperature is concrete: it is the result of all physical processes involved in the heat of physical bodies (or lack thereof). In contrast, the mathematical representation of temperature in terms of real numbers (or any other mathematical structure) clearly requires idealizations and the introduction of abstract patterns. But this is a feature of the mathematical representation (the representation cannot be implemented without a proper mathematical scale) rather than of the process being represented. But since the physical process that one aims to represent is a spatiotemporal phenomenon, it is concrete, not abstract.

Moreover, note that the idealization described in the objection above yields two structures: one that represents measurable differences in warmth of physical objects, and another that provides the scale that will be used to measure such differences (in this case, real numbers). A mapping between these two structures is then established. Interestingly, the former structure is clearly partial (since several relations among the relevant physical objects are not specified), and the relevant mapping between the two structures is a partial one (a partial homomorphism, in this case). So even the idealized representation of temperature can be properly captured via partial mappings in the end.

5 Conclusion

For the reasons discussed above, I think structuralism can solve the 'access' problem, without requiring any access to mathematical objects. This indicates a significant feature of mathematical structuralism: it's a philosophical position in which the objects mathematical theories describe – to the extent that these objects are so described – do not play a role in how we get to know these objects (see also AZZOUNI, 1994). As a result, structuralism provides a fundamentally different picture of mathematical epistemology than traditional epistemology would suggest.

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