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SEÇÃO: ARTIGOS

REVISITING THE THEORETICAL FOUNDATIONS OF ETHNOMODELLING

REVISITANDO AS FUNDAMENTAÇÕES TEÓRICAS DA ETNOMODELAGEM

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Abstract: By including mathematical activities from outside of the school environment, the process of modelling shows us that mathematics is more than the manipulation of mathematical symbols, procedures, and practices. The application of ethnomathematical techniques along with the tools of modelling allows us to see a holistic reality to mathematics. From this perspective, one pedagogical approach that connects the cultural features of mathematics with its school/academic aspects is named ethnomodelling, which is a process of translation and elaboration of problems and questions taken from systems that are part of the reality of the members of any cultural group. In this article we offer an alternative goal for educational research, which is the acquisition of both emic (local) and etic (global) approaches for the implementation of ethnomodelling in the classrooms. We also discuss a third approach on ethnomodelling, which is the dialogical (glocal) approach, which combines both emic and etic approach bases. Finally, we define ethnomodelling as the study of mathematical phenomena within a culture because it is a social construct and is culturally bound, which adds the cultural characteristics of mathematics into the modelling process.

Keywords: ethnomathematics; ethnomodelling; mathematical modelling; sociocultural perspective.

Resumo: Ao incluir as atividades matemáticas fora do ambiente escolar, o processo de modelagem nos mostra que a Matemática é mais do que a manipulação de símbolos, procedimentos e práticas matemáticas. A aplicação de técnicas etnomatemáticas juntamente com as ferramentas de modelagem permitem ver uma realidade holística para a Matemática. Nessa perspectiva, uma abordagem pedagógica que conecta os aspectos culturais da Matemática com os seus aspectos escolares/acadêmicos é denominada etnomodelagem, que é um processo de tradução e elaboração de problemas e questões retirados de sistemas que fazem parte da realidade dos membros de qualquer grupo cultural. Neste artigo, oferecemos um objetivo alternativo para a pesquisa educacional, que é a aquisição de abordagens êmicas (locais) e éticas (globais) para a implementação da etnomodelagem em salas de aula. Discutimos também uma terceira abordagem para a etnomodelagem, que é a abordagem dialógica (glocal), que combina as abordagenes êmica (local) e ética (global). Finalmente, definimos a etnomodelagem como o estudo de fenômenos matemáticos dentro de uma cultura porque é um construto social e culturalmente vinculado que adiciona as características culturais da Matemática ao processo de modelagem.

Palavras-chave: etnomatemática; etnomodelagem; modelagem matemática; perspectiva sociocultural.

INITIAL CONSIDERATIONS

Ethnomathematics arises when researchers investigate the knowledge possessed by members of distinct cultural groups in the context of their mathematical ideas, techniques, procedures, and practices. However, an outsiders' understanding of cultural traits remains an interpretation that

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may come to emphasize inessential features of the culture and/or is in danger of creating a misinterpretation of a unique and culturally specific mathematical paradigm.

The challenge that arises from this approach is how culturally bound mathematical ideas can be extracted or understood without letting the culture of the researchers (outsiders) interfere with their findings, interpretation, and discussions related to the mathematical knowledge of the members of the cultural group under study. This fact may happen when the members of distinct cultural groups develop the interpretation of their own culture, which is named an *emic* (local) approach as opposed to an outsiders' interpretation, which is named the *etic* (global) approach.

The concepts of emic (local) and etic (global) were first introduced by the linguist Pike (1954) who drew upon an analogy regarding two linguistic terms:

a) Phon***emic***, which is considered as the specific sounds used in a particular language.
b) Phon***etic***, which is considered as the general aspects of vocal sounds and sound production in a particular language.

In this context, we consider that all the possible sounds human beings can make constitute the phonetics of a given language. However, when people speak a particular language, they do not hear all its possible sounds. In this regard, not all sounds make a difference when spoken because they are locally significant. This means that they are the phonemics of that language.

Researchers and educators who take on an emic (local) approach consider that many factors such as cultural and linguistic backgrounds, social, moral values, and lifestyle come into play when mathematical ideas, procedures, and practices are developed by members of their own culture. Thus, these members have developed different ways of doing mathematics in order to understand and comprehend their own cultural, social, political, economic, and natural environments (ROSA, 2010).

Furthermore, members of distinct cultural groups have developed unique and often very distinct ways to *mathematize* their own realities (D'AMBROSIO, 1990). In this context, mathematization is the process in which individuals from distinct cultural groups come up with different mathematical tools that can help them to organize, analyze, comprehend, understand, solve, and model specific problems located in the context of their own real-life situation (ROSA; OREY, 2006).

These tools allow them to both identify and describe specific mathematical ideas, procedures, or practices in a general context by schematizing, formulating, and visualizing a problem in different ways, discovering relations and regularities, and transferring a real world problem to academic mathematics through mathematization. In this context, ethnomodelling is considered one alternative methodological approach that enables us to record historical forms of mathematical ideas, procedures, techniques, and practices that occur in distinct cultural contexts.

Consequently, Rosa and Orey (2010a) define ethnomodelling as the practical application of ethnomathematics that adds a cultural perspective to the modelling process. Thus, when justifying the need to the development of a culturally bound view on modelling, our sources are rooted on the theorical basis of ethnomathematics and on the sociocultural perspective of mathematical modelling.

ETHNOMATHEMATICS

Ethnomathematics was introduced by the Brazilian educator and Mathematician Ubiratan D' Ambrosio in the 1970s. Ethnomathematics uses the etymology of three Greek roots: *ethno*, *mathema*, and *tics* (ROSA; OREY, 2016). It is a program that incorporates mathematical ideas and procedures practiced by the members of distinct cultural groups, which are identified not only as indigenous societies but as groups of workers, professional classes, and groups of children of a certain age as well (D'AMBROSIO, 1990).

Thus, *ethno* refers to members of a specific group within a cultural environment identified by their cultural traditions, codes, symbols, behaviors, myths, and specific ways used to reason and to infer. *Mathema* means to explain and understand the world to transcend, manage, and cope with reality so that the members of the cultural groups can survive and thrive in their daily endeavors and *tics* refers to techniques, procedures, and strategies such as counting, ordering, sorting, measuring, weighing, ciphering, classifying, inferring, and modelling (D'AMBROSIO, 1993).

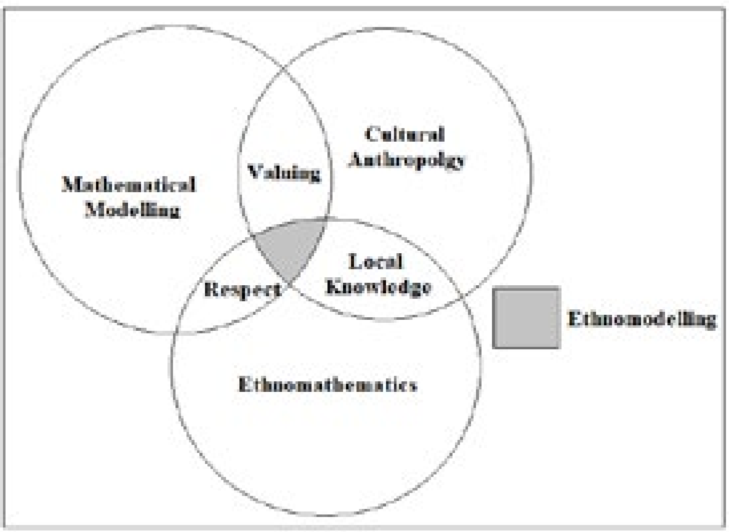
Ethnomathematics as a research paradigm is wider than traditional concepts of mathematics, ethnicity, or any current sense of multiculturalism. Ethnomathematics is described as the arts and techniques (*tics*) developed by individuals from diverse cultural and linguistic backgrounds (*ethno*) to explain, to understand, and cope with their own social, cultural, environmental, political, and economic environments (*mathema*) (D'AMBROSIO, 1990). *Ethno* refers to distinct groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring.

Detailed studies of mathematical procedures and practices of distinct cultural groups most certainly allow us to further our understanding of the internal logic and mathematical ideas of diverse groups of people. For example, Rosa (2010) affirmed that principles of anthropology relevant to the work of ethnomathematics include the essential elements of culture such as language, economy, politics, religion, art, and the daily mathematical practices of diverse groups of students.

Since cultural anthropology gives us tools that increase our understanding of the internal logic of the members of a given cultural group; detailed anthropological studies of the mathematics of distinct cultures most certainly allows us to further our understanding of the internal logical system and beliefs of diverse group of students. Thus, we consider ethnomathematics as the intersection of cultural anthropology, mathematics, and mathematical modelling, which is used to help us understand and connect diverse mathematical ideas and practices found in our communities to traditional and academic mathematics.

Consequently, Rosa and Orey (2010b, p. 60) consider ethnomodelling as the intersection of three research fields: cultural anthropology, ethnomathematics, and mathematical modelling, which can be used "as a tool towards pedagogical action of an ethnomathematics program, students have been shown to learn how to find and work with authentic situations and real-life problems". Figure 1 shows ethnomodelling as the intersection of 3 (three) research fields: cultural anthropology, ethnomathematics, and mathematical modelling.

Figure 1 – Ethnomodelling as the intersection of three research fields



Source: Adapted from Rosa and Orey (2010).

According to Rosa and Orey (2010a), cultural anthropology studies distinct cultures and how their members shape the world around them by studying similarities and differences regarding the development of mathematical procedures and techniques. The goal for education is to learn how to collect data on how political, economic, social, environmental, and cultural practices to understand how mathematics is influenced by the cultures that are studied.

Ethnomathematics seeks to study how students have come to understand, comprehend, articulate, process, and ultimately use mathematical ideas, procedures, and practices that enable them to solve problems related to their daily activities. This helps students to reflect, understand, and comprehend extant relations among all components of systems under study. In this regard, educators should be empowered to analyze the role of students' *ethnoknowledge* in the mathematics classroom, which is acquired by students in the process of pedagogical action of learning mathematics in culturally relevant educational systems (ROSA, 2010).

Ethnomathematics researchers investigate ways in which members of distinct cultural groups comprehend, articulate, and apply ideas, procedures, and techniques identified as mathematical practices. In this regard, Rosa and Orey (2003) affirmed that ethnomathematics uses cultural experiences as vehicle to make mathematics learning more meaningful and to provide students with the insights of mathematical knowledge as embedded in their social and cultural environments.

Ethnomathematics empowers students intellectually, socially, emotionally, and politically by using their sociocultural and historical realities and contexts to convey knowledge, impart academic skills, and change students' attitudes towards academic instruction (ROSA; OREY, 2012). According to Barton (1996), ethnomathematics embraces the mathematical ideas, thoughts, concepts, procedures, and practices as developed by the members of all cultures. From this perspective, a body of anthropological research has come to focus on both the intuitive mathematical thinking and the cognitive process that are largely developed in distinct cultural groups.

Following the above definition, Ethnomathematics may also be considered as a program that seeks to study how students have come to understand, comprehend, articulate, process, and ultimately use mathematical ideas, concepts, procedures, and practices that may solve problems related to their daily activities. Ethnomathematics program and its pedagogical action of teaching and learning mathematics that is built on students' *tacit knowledge*[2], background, the role his environment plays in terms of content, methods, and his past and present experience of his immediate environment (ROSA, 2010).

Ethnomathematics as applied in a school context involves taking students to situations that allow them to actively participate in culturally influenced activities in the classroom, such as house roofing, traffic, prices in a supermarket or neighborhood outdoor vegetable market, and the representation of models of real-life situations in educational environments for purposes of interaction between the learner and materials to achieve educational objectives (ROSA; OREY, 2016).

In this regard, the arguments often given for using ethnomathematical examples in classrooms are: (a) to show students of underrepresented cultures that their own cultures do contribute to mathematical thinking and (b) to expose students of majority cultures to diverse cultures from around the world, building respect for others and generally contributing to global education. These are certainly laudable goals, however, occasionally ethnomathematicians have expressed concern that too often Western field research tends to seek the *others* to the extent of exploiting

[2] Tacit knowledge is the unwritten, unspoken, and hidden knowledge held by members of distinct cultural groups, which is based on their emotions, experiences, insights, intuition, observations, and internalized information developed through the resolution of phenomena they face in their daily life. It is integral to the development of the consciousness of these members because it is acquired through association with members of *other* cultural groups and requires joint or shared activities to be imparted from one to another. It constitutes a set of informally developed knowledge and forms the underlying framework that makes explicit knowledge possible (ROSA; OREY, 2012).

indigenous cultures (ROSA; OREY, 2016) without giving them the voice that allows them to share and explain how they actually do the math.

In this context, Gavarrete (2014) states that one possible way to avoid this problem and, notably, bring the goals of ethnomathematics even more directly to students, is to encourage students to develop ethnomathematical studies of their own individual cultures, heritage, and personal interests. Therefore, if students make presentations to each other, they all learn about all the cultures represented in the classrooms, not just the one expressed by the formal curriculum and the textbook. Students from underrepresented groups can demonstrate the contributions of their cultural group.

Mathematical knowledge is perceived in an ethnomathematical perspective because teachers build on the students' informal mathematics and direct the lesson toward their culture and experiences while developing their critical thinking skills. This environment enables us to reflect on the nature of mathematics, culture, education, and society and the relationships among them in order to include pedagogical practices in the teaching and learning of mathematics that address deeper notions of equality and equity (GAVARRETE, 2014).

Examples can come from family traditions, hobbies, religions, and occupations; geography-based activities; celebrations of holidays and life events; personal interests such as sports, music, art, dance, or crafts; and even child-related activities, from playground games and computer games to skateboarding, jumping rope, and birthday parties. All bring the students' attention to cultures, and all show applications of mathematics in context (GAVARRETE, 2014).

It is necessary for teachers to be supported to allow them to emphasize the connections between mathematics and other curricular disciplines and consider students' cultural backgrounds in designing and selecting mathematical activities. Students learn in ways characterized by social and affective approaches, harmony with the community, holistic perspectives, field dependence, expressive creativity, and non-verbal communication (ROSA, 2010).

This context enables the evolution of ethnomathematics as a research field in which one of the main goals is to link local (emic) knowledge to the mathematics curriculum (global, etic) by applying innovative approaches to mathematics through dialogue (glocal, cultural dynamism) (ROSA; OREY, 2017).

It is important to discuss interrelated innovative approaches in ethnomathematics programs, such as their relation to social justice, civil rights, indigenous education, professional contexts, game playing, urban and rural contexts, ethnotransdisciplinarity, ethnopedology, ethnomethodology, ethnomodelling, ethnocomputing, and the Trivium Curriculum (ROSA; OREY, 2017).

It is important to emphasize that researchers conducting ethnomathematical invetigations study the cultural congruence between the backgrounds of students, communities, and schools, which in turn form one of the main principles of an ethnomathematics program. An important characteristic of ethnomathematics is its transformational power, and how it can help to rethink the nature of mathematics (THOMAS; HART, 2010).

One possible purpose for ethnomathematical studies, and its many innovative approaches, could be to foster the development or transformation of mathematics itself (D'AMBROSIO, 1993). To remain relevant, mathematics instruction needs to accommodate continuous and ongoing changes in students' demographics in mathematics classrooms around the world. Since ethnomathematics proposes that educators contextualize their mathematics teaching and learning by relating the content to sociocultural experiences of their students (ROSA, 2010).

Hence, it has become necessary to integrate diverse ethnomathematical perspectives into existing teacher education programs and encourage students to examine mathematical activities in their own sociocultural contexts. To help them to see, indeed realize how mathematics procedures and practices are not trivial and help them to connect what they are learning to their daily lives. In this perspective, students may succeed in mathematics when their understanding of it is linked to real and meaningful cultural referents and when instruction assumes that all students are capable of mastering mathematics (ROSA; OREY, 2007).

Ethnomathematics presents possibilities for educational initiatives and innovative curriculum objectives based on an ethnomathematical perspectives. However, one dilemma regarding this issue is related to how we can prepare teachers to create curriculum activities based on ethnomathematics. One important approach to solve this dilemma is to focus on the importance of promoting the dissemination of heritage aspects of local (emic), cultural, and mathematical knowledge to help students strengthen their own cultural identities in school environments (GAVARRETE, 2014).

In this regard, Rosa and Orey (2003) affirmed that ethnomathematics uses cultural experiences as vehicle to make mathematics learning more meaningful and to provide students with the insights of mathematical knowledge as embedded in their social and cultural environments. Ethnomathematics contributes to restoring cultural dignity and offers the intellectual tools for the exercise of a citizen.

In accordance with D'Ambrosio (1993), it enhances creativity, reinforces cultural self-respect, and offers a broad view of mankind. In everyday life, it is a system of knowledge that offers the possibility of more favorable and harmonious relation between humanity and nature. In this approach, ethnomathematics aims at drawing from the learners' cultural experiences and practices of the individual learners, the communities, and the society at large.

Ethnomathematics uses cultural experiences as vehicles to make mathematics learning more meaningful and to provide students with the insight of mathematical knowledge as embedded in their social and cultural environments (ROSA; OREY, 2012). Ethnomathematics presents mathematical concepts of the school curriculum in such a way that they are related to students' cultural and daily experiences, it creates a sense of relevance, whereby enhancing meaningful connections and deepening their understanding in mathematics. Ethnomathematics as part of the school curriculum must reinforce and value the cultural knowledge of students rather than ignore or negate it.

Every culture has its own way of mathematizing concepts which are part of its inheritance and the result of the struggle for its survival (D'AMBROSIO, 1990). Throughout history, Mathematics has been used by different people in many ways. Egyptians used geometry to construct pyramids for burial purposes. Therefore, ethnomathematics may be defined as how people of various cultures use mathematics in their everyday life (ROSA; OREY, 2017).

Thus, it is necessary to highlight that the Trivium Curriculum for mathematics proposed by D' Ambrosio (1993) as an important, indeed innovative, ethnomathematics approach that needs more investigation in order to address pedagogical purposes, as well as it helps to reach the proposed objectives of ethnomathematics. The overall goal of this curriculum is to empower students through learning activities that help them develop literacy, numeracy, material and technological, social, and political skills in order for them to be active participants in a democratic society.

SOCIOCULTURAL PERSPECTIVE OF MATHEMATICAL MODELLING

Mathematical modelling constitutes one of the most important research trends for the development of teaching and learning processes in mathematics (ROSA, 2010). In this regard, it is important to point out that this pedagogical action is directed towards the resolution of everyday problems and situations, with the use of modelling to encourage educators and learners to value and enable the connection between mathematics and the daily experiences lived by members of distinct cultural groups.

In this context, Bassanezi (2002) states that mathematical modelling is the art of transforming reality problems into mathematical problems and solving them by interpreting their solutions in the language of the real world. According to this approach, Rosa (2010) affirms that modelling techniques provide the contextualization of academic school/mathematics by providing necessary conditions to the development of pedagogical actions through the elaboration of mathematical models, so that the members of these cultural groups can act satisfactorily in the *glocalized world*[3].

Accordingly, mathematical modeling presupposes the use of multidisciplinary approaches because it has confluences with other trends in mathematics education. Ethnomathematics, which, such as point to the removal of boundaries between the various areas of research (BASSANEZI, 2002). Thus, there is a need to consider mathematics education as a scientific field directed towards to the teaching and learning process in mathematics through its resignification, which enables the understanding and perception of its importance in the development of this pedagogical action (ROSA, 2010).

Similarly, ethnomodelling relates the diversity of concepts inherent to ethnomathematics with mathematical modelling (ROSA; OREY, 2010b). For example, Caldeira (2007) states that it is necessary to consider mathematics constructed and signified in the cultural practices developed by the members of local communities, as well as the many influences of these meanings in the pedagogical process in order to apply mathematical modelling as a means to achieve the objectives proposed for the conduction of this pedagogical action through its complementarity with ethnomathematics.

However, for this objective to be achieved, Rosa and Orey (2012) argue that there is a need for students to be inserted in a learning environment that enables them to use mathematical knowledge that was previously acquired at school and developed and accumulated in the community in which they are inserted.

Historically, models that originate in the reality of members of distinct cultural groups can be considered as pedagogical tools that are used for the abstraction of mathematical concepts, because member of distinct cultural groups develop its own set of ideas and mathematical concepts, among which some basic tools that are used in the development of the modelling process stand.

These tools can be understood as the ways that members of each cultural group develop methods that allow them to deal with their own realities and to mathematize and model their world through the use of measurement, comparison, quantification, classification, and inference often uniquely developed locally.

From the perspective of Cortes (2017), this context allows the exploration of ideas, procedures, and local mathematical practices, which aims to value and respect diverse cultural values and the knowledge acquired by students through their own experiences in society. Therefore, modelling is an important tool to help students to understand, comprehend, analyze, and reflect on their own sociocultural contexts. In this regard, being proficient in the use of modeling is of fundamental importance so that members of distinct cultural groups, through their actions, modify their own reality so that they can be included in the process of social transformation in a critical and reflective way (ROSA; OREY, 2017).

For example, Rosa and Orey (2009) state that, through the modeling process, it is possible to show that a key aspect of this process is to help students realize their mathematical potential through the recognition of the importance of culture for the appreciation of their own identity because this aspect influences the way they think, learn, reflect, infer, and takes informed decisions.

Thus, Rosa and Orey (2012) state that mathematical modelling is a learning environment that facilitates the construction and transfer of mathematical knowledge through the use of their mathematical knowledge: a) explicit[4] and tacit, which interact in this environment. In this

[3] A glocalized world enables the development of active, interactional, and dialogical processes that requires an ongoing negotiation between the local and the global mathematical, scientific, technological, and engineering knowledge through the development of a cultural dynamism (ROSA; OREY, 2017).

[4] Explicit knowledge is related to a concrete fact, which can be disseminated by teachers through the use of textbooks, academic

context, members of distinct cultural groups have developed and are developing diverse and different ways of doing mathematics. Thus, D'Ambrosio (1990) states that members of these groups have developed, throughout history, distinct ways to mathematize their own reality by using elements of the modelling process.

Therefore, mathematization is the process through which members of distinct groups to use different mathematical tools to help them organize, analyze, understand, understand, model, and solve the problems faced in their daily lives (ROSA; OREY, 2006). These tools enable, according to Rosa and Orey (2017), the identification of ideas in order to describe procedures and mathematical practices specific to a cultural context, which aim to help these members to discover relationships and regularities.

Hence, highlight that this cultural approach to modeling allows these members to schematize, formulate, and visualize problems and situations in different ways, which help them transcend the solution of real-world phenomena to mathematical conceptualization through the mathematization process (ROSA; OREY, 2003).

In this direction, Rosa and Orey (2017) state that the sociocultural perspective of mathematical modelling involves the study of mathematical ideas, procedures, and practices that are found in different cultural contexts so that they can be used in their pedagogical action in classrooms through the elaboration of ethnomodels.

Thus, mathematical modelling procedures can be employed when ethnomathematics is actively used as a system based on a theoretical basis that helps members of distinct cultural groups in the development of solving everyday problems related to the social, cultural, economic, political, and environmental contexts (ROSA; OREY, 2017).

The connection between mathematical modelling and the cultural aspects of mathematics through ethnomathematics is considered as the study of ideas and procedures used in mathematical practices, which were developed by members of different cultural groups when considering the acquired mathematical knowledge and cultural practices in the community with the use of problem-solving techniques developed locally (ROSA; OREY, 2010b).

These techniques are considered as the basic tools used by ethnomodelling that help teachers and researchers in carrying out the translation between the emic and etic approaches (ROSA; OREY, 2016). Thus, ethnomodelling is a tool that aims to mediate the cultural forms of mathematics with the school curriculum to facilitate the development of its teaching and learning process.

According to this context, mathematical modelling is a teaching trend in mathematics education that aims to develop critical and reflective students who are aware of the different problems that are faced in their everyday life. However, for this objective to be achieved, there is a need for students to be inserted in a learning environment that allows them to use of mathematical knowledge previously acquired at school and tacitly in the community in which they are inserted. This approach will help students to contextualize curricular activities in the daily life of the students (ROSA; OREY, 2007).

From this perspective, Rosa and Orey (2017) comment that modelling techniques provide the contextualization of school/academic mathematics by providing the necessary conditions through the elaboration of mathematical models, so that members of distinct cultural groups can act in the glocalized world. For example, Rosa and Orey (2007) state that this contextualization is an important concept for the development of citizens students, as it offers an opportunity for the teaching of sociocultural efficiency[5].

In this regard, teachers have the responsibility

approach about the subject, knowledge of pedagogical instructional practices, and any other method of using materials and technological instruments that can to help them to absorb, internalize and, consequently, transfer and diffuse the applicability of this knowledge to other areas of human knowledge (ROSA; OREY, 2012).

5 The fundamental characteristic of sociocultural efficiency is the emphasis on students' critical analysis of society's power structures. Another important feature is the students' personal reflection on the social elements that underpin the globalized world. Thus, the critical perspective of students in relation to the social conditions that affect their own experiences can help them to identify common problems and, collectively, develop different strategies to solve them (ROSA; OREY, 2007).

to favor the establishment of relationships between school/academic mathematics and students' tacit knowledge, so that they can perceive the presence of mathematics in the activities they experience daily (ROSA; OREY, 2009). Thus, it is necessary that, in the classrooms, teachers discard the traditional passive and transmissive pedagogical models and favor the transformative pedagogical model.

Thus, Rosa and Orey (2007) state that the traditional teaching method predominant in the educational system tends to focus on the traditional objective of learning for the transmission of mathematical knowledge. So, it is necessary to discard this traditional model so that sociocultural efficiency in education is implemented in the classrooms.

In this context, agreeing with the point of view of Rosa and Orey (2012) who present modelling as a learning environment, in which teachers and students are responsible for the development of mathematical knowledge and for the conversion between its tacit and explicit dimensions, from situations arising, preferably, from their own realities.

From the perspective of Rosa and Orey (2007), the conception of the role of students in this approach is that of active collaborators in the learning process, which is a more stimulating task than the one related to the simple reception of mathematical knowledge and practices. In sociocultural mathematical modelling, students can be considered as creators of mathematical knowledge, as this process provides the conditions for them to get involved with mathematics, so that they can challenge it, understand it, and interpret it by making it into a product of human creation.

In this direction, Cortes (2017) states that learning is triggered according to the students' purpose, as it develops differentiated capabilities so that they can act, react, reflect, and change the environment in which they live by transforming it, strategically. Thus, this environment influences the development of students' cognitive process in different ways, as it is related to their sociocultural context.

Therefore, the sociocultural dimension of mathematical modelling has as background the social and cultural knowledge theories, which are related to the emancipatory perspective and transformative learning that apply the philosophical ideals of the critical thinking theory (ROSA; OREY, 2007). Therefore, sociocultural theory is related to learning processes that are triggered through socialization, as knowledge is better constructed when students interact to socialize learning.

Thus, students act cooperatively and collaboratively to support and encourage each other, so that they can reflect on the resolution of complex problems rooted in authentic situations (ROSA; OREY, 2007). As in the mathematical modelling process, it is important that students actively participate in the construction of their mathematical knowledge by connecting it interdisciplinary with other areas of knowledge in an interdisciplinary fashion (ROSA; OREY, 2017).

In the sociocultural theory, Rosa and Orey (2007) affirm that the joint work between teachers and students makes learning more effective, because cultural tools, such as artifacts, language, traditions, behaviors, and institutions are shared. So, the meaning of learning is constructed in the social context, as members of different cultural groups learn together and collaboratively through integrated experiences.

Thus, it is necessary for students to develop their abilities to solve problems, make decisions, work in teams, and communicate effectively, all of which are important characteristics of sociocultural mathematical modelling. For example, Bassanezi (2002) states that the analysis of data through statistics and the interpretation of results determined in studies have contributed to direct the use of action strategies in commercial, social, environmental, and political contexts.

Consequently, Rosa and Orey (2007) argue about the need to apply the notions of the emancipatory approach to mathematical modelling, whose educational objectives address issues of a sociopolitical nature and their consequences in the pedagogical practices used in school systems. For example, Rosa and Orey (2007) claim that this emancipatory approach is based on the sociocultural competence of members of distinct cultural groups in which its main objective is to help students to face and solve challenges imposed by the globalized society.

Thus, this approach must be directed to transform students into flexible, adaptable, reflective, critical, and creative citizens using alternative pedagogical methodologies that aims to value and record ideas, procedures, and mathematical practices that are developed in distinct cultural contexts (ALVES, 2014). Therefore, Rosa and Orey (2007) state that the sociocultural aspect of modelling is based on the expansion of students' autonomy, as it aims to provide a critical reading of their worldview, as well as for the development of their autonomous thinking, which aims to contribute to the full exercise of their citizenship.

DISCUSSING ETHNOMODELLING

Ethnomodelling is the study of mathematical ideas and procedures elaborated by members of distinct cultural groups. It involves the mathematical practices developed, used, practiced, and presented in diverse situations in the daily life of the members of these groups (ROSA; OREY, 2010a).

This context is holistic and allows those engaged in this process to study mathematics as a system taken from their own contextual reality in which there is an equal effort to create an understanding of all components of these systems as well as the interrelationship among them (D'AMBROSIO, 1993; BASSANEZI, 2002; ROSA; OREY, 2003).

Investigators and educators such as Ascher (2002), Eglash (1999), Orey (2000), Urton (1997), and Rosa and Orey (2009) "have revealed [in their research] sophisticated mathematical ideas and practices that include geometric principles in craft work, architectural concepts, and practices in the activities and artifacts of many indigenous, local, and vernacular cultures" (EGLASH *et al.*, 2006, p. 347). These concepts are related to the numeric relations found in measuring, calculation, games, divination, navigation, astronomy, modelling, and a wide variety of other mathematical procedures and cultural activity (EGLASH *et al.*, 2006).

Researchers such as Eglash *et al.* (2006) and Rosa and Orey (2006) use the term *translation* to describe the process of modelling local cultural systems (emic/local), which may have a Western school/academic mathematical representation (etic/global). This context allows for the translation of interpretations and contributions of ethnomathematical knowledge into systemized mathematics as students learn to construct their own connections between both traditional (global/etic) and non-traditional (local/emic) learning settings through translations and symmetrical dialogues.

In this regard, ethnomathematics makes use of modelling by attempting to use it to establish relations between the local conceptual framework (emic/local) and the mathematics embedded in relation to local designs. On the other hand, many indigenous designs, such as the applications of symmetry classifications from crystallography to indigenous textile patterns, have been analyzed from a Western view (etic/global).

In some cases, Eglash *et al.* (2006, p. 347) state that "the translation to Western mathematics is direct and simple such as counting systems and calendars". However, there are cases in which mathematical ideas and concepts are "embedded in a process such as iteration in bead work, and in Eulerian paths found in sand drawings" (EGLASH *et al.*, 2006, p. 348). Thus, the act of translation applied in this process is best referred to as ethnomodelling. In this process "mathematics knowledge can be seen as arising from emic rather than etic origins" (EGLASH *et al.*, 2006, p. 349).

This means that the mathematics we use in academic/scientific contexts was not conceived as a universal language because its principles, concepts, and foundations were not the same everywhere (ROSA, 2010). In this regard, any choice among equivalent representational systems can only be founded on considerations of simplicity, for no other consideration can adjudicate between equivalent systems that univocally designate reality (CRAIG, 1998).

In this context, Rosa and Orey (2006) affirm that the processes of production of mathematical ideas, procedures, and practices operate in the register of the interpretative singularities regarding the possibilities for a symbolic construction of knowledge in different cultural groups. This context allows for the translation of interpretations and contributions of ethnomathematical knowledge into systemized mathematics as students learn to construct their own connections between both traditional and non-traditional learning settings through translations and symmetrical dialogue.

The Emic and Etic Constructs of Ethnomodelling

In the ethnomodelling approach, the emic constructs are the accounts, descriptions, and analyses expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the members of the cultural group under study. This means that an emic construct is in accordance with the perceptions and understandings deemed appropriate by the insider's culture. The validation of emic approach comes with a matter of consensus of local people who must agree that these constructs match the shared perceptions that portray the characteristic of their culture (LETT, 1996).

In other words, the emic approach investigates mathematical phenomena and their interrelationships and structures through the eyes of the people themselves. It is important to note that research techniques used in acquiring emic mathematical knowledge have nothing to do with the nature of that knowledge. In this regard, "emic mathematical knowledge may be obtained either through elicitation or observation because it is possible that objective observers may infer local perceptions" (LETT, 1996, p. 382) about mathematical ideas, procedures, and practices.

It is necessary to state that etic constructs include individual accounts, descriptions, and analyses of mathematical ideas, concepts, procedures, and practices expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers, researchers, and investigators (LETT, 1996). An etic construct is precise, logical, comprehensive, replicable, and observer-researcher independent.

Any validation of etic approach thus becomes a matter of logical and empirical analysis, in particular, contains the logical analysis of whether the construct meets the standards of comprehensiveness and logical consistency, and then the empirical analysis of whether or not mathematical concepts have been replicated (LETT, 1996).

It is important to emphasize that particular research techniques used in the acquisition of etic mathematical knowledge has no bearing on the nature of that knowledge. The etic approach may be obtained at times through elicitation as well as observation (LETT, 1996). According to D'Ambrosio (1990), investigators and educators must acknowledge and recognize that local people possess scientifically and mathematically valid knowledge.

Dialogical Approach of Ethnomodelling

If we make an analogy in regard to ethnomodelling, it is possible to state that its emic (local) approach is concerned about differences that make mathematical practices unique from an *insider's* point of view. We argue that emic ethnomodels are grounded in what matters in the mathematical world of those being modeled. On the other hand, many ethnomodels are etic in the sense that they are built on an outsider's observation of *the others* about the world being modeled.

In this context, etic (global) ethnomodels represent how modelers themselves think the world works through systems taken from reality while emic ethnomodels represent how people who live in such worlds think these systems work in their own reality. We also would like to point out how etic approaches play important roles in ethnomodelling research, yet the emic approach should be also taken in consideration.

In this perspective, the emic ethnomodels

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sharpen the question of what an agent-based model should include to serve practical goals in modelling research. Thus, mathematical ideas and procedures are etic if they can be compared across cultures using common definitions and metrics while the focus of the analysis of these aspects are emic if the mathematical ideas, concepts, procedures, and practices are unique to a subset of cultures that are rooted on the diverse ways in which etic activities are carried out in a specific cultural setting.

Currently, the debate between emic-etic is one of the most intriguing questions in ethnomathematics and mathematical modelling research since researchers continue to explore questions such as:

1. Are there mathematical patterns that are identifiable and/or similar across cultures?
2. Is it better to focus on these patterns particularly arising from the culture under investigation?
3. How does data gained from outsiders differ from that of the insiders in relation to emic (local)-etic (global) research data?

Emic (local) and etic (global) approaches are often thought of as creating conflicting dichotomies. For example, Pike (1954) originally conceptualized them as complementary viewpoints. According to this context, rather than posing a dilemma, the use of both approaches deepens our understanding of important issues in scientific research and investigations (BERRY, 1999). A suggestion for dealing with this dilemma is to use a combined emic-etic approach, rather than simply applying emic or etic dimensions of one culture to other cultures.

A combined emic-etic approach requires researchers to first attain emic approach about the cultural groups under study. This may allow them to become aware of and then put aside cultural biases, and to become familiar with the relevant cultural differences in each setting (BERRY, 1999).

Usually, in ethnomodelling research, an emic analysis focuses on a single culture and employs descriptive and qualitative methods to study a mathematical idea, concept, procedure, or practice of interest. Its focus is on the study within the cultural group context in which the researcher tries to develop research criteria relative to internal characteristics or logic of the cultural system. In this perspective, meaning is gained relative to the context and therefore not easily, or of at all transferable to other contextual settings.

For example, it is not intended to compare the observed mathematical patterns in one setting with mathematical patterns in other settings. This means that the primary goal of an emic approach is a descriptive idiographic orientation of mathematical phenomena because it puts emphasis on the uniqueness of each mathematical idea, procedure, or practice developed by the members of cultural groups.

Thus, if researchers and educators wish to highlight meanings of these generalizations in local or emic ways, then they need to refer to more precisely specified mathematical events. In contrast, an etic analysis would be comparative, examining many distinct mathematical cultural practices by using standardized methods (LETT, 1996). This means that the etic approach tries to identify lawful relationships and causal explanations valid across different cultures. Thus, if researchers and educators wish to make statements about universal or etic aspects of mathematical knowledge, these statements need to be phrased in abstract ways.

On the other hand, an etic (global) approach may be a way of getting at the emics of the members of cultural groups because it may be useful for penetrating, discovering, and elucidating emic (local) systems that were developed by members of different cultural groups (PIKE, 1954). In so doing, while traditional emic and etic concepts are important points of view for understanding and comprehending cultural influences on mathematical modelling, we would like to propose a distinctively different view on ethnomathematics and modelling research, which is referred as a *dialogical approach* (MARTIN; NAKAYAMA, 2007).

In this approach, the etic approach claims that the knowledge of any given cultural group has no priority over its competing emic claims. According to this point of view, it is necessary to depend "on acts of 'translation' between emic and etic approaches" (EGLASH *et al.*, 2006, p. 347). In other words, the cultural specificity may be better understood with the background of communality and the universality of theories and methods and vice versa.

In this context, these insights must be verified with methods independent of the subjectivity of the observer and researcher to achieve a scientific character. In so doing, it is important to analyze the insights that have been acquired through subjective and culturally contextualized methods. The rationale behind the emic-etic dilemma is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur.

Mathematical Phenomena and Ethnomodels

Through history, many researchers, and investigators made extensive use of mathematical procedures ranging the encountered as Europeans colonized and extended their empires through colonial trade connections. The diverse procedures they encounters originated from geometrical and statistical methods, methods for the elucidation of patterns in behavior, and to the mathematical representations and logic processes of indigenous conceptual systems they encountered for better or worse (ROSA; OREY, 2017).

Mathematical modelling has been considered by some metaphorically as a pedagogical tool and by others as a way to understand anthropological and archaeological research. Yet, others have decried the use of mathematical, and in particular, statistical and quantitative modelling as fundamentally in opposition to a humanistic approach to understanding human behavior and knowledge that takes into account contingency and historical embeddedness and in turn, decries universality.

However, we argue that traditional mathematical modelling does not fully consider the implications of cultural aspects of human social systems. It also serves to introduce learners to increasingly powerful and formal mathematical knowledge needed to resolve more and more complex problems through formal modelling.

The cultural component in this process is critical because its accounts "emphasize the unity of culture, viewing culture as a coherent whole, a bundle of [mathematical] practices and values" (POLLAK; WATKINS, 1993, p. 490) that are incompatible with the rationality of the elaboration of traditional mathematical modelling process. However, in the context of mathematical forms of knowledge, what is meant by the cultural component, varies widely and ranges from viewing mathematical practices as learned and transmitted to the members of cultural groups to formal academic practices viewed as made up of abstract symbolic systems with an internal logic giving a symbolic system its mathematical structure.

If the former is considered, then it is the process by which knowledge transmission takes place from one person to another, which is central to elucidating the role of culture in the development of mathematical knowledge (D'AMBROSIO, 1993). If the latter is considered, then culture plays a far more reaching and constructive role with respect to mathematical practices that cannot be induced simply through observation of these practices (EGLASH *et al.*, 2006).

In this regard, mathematical knowledge developed by members of a specific cultural group consists of abstract symbol systems whose form is the consequence of a unique internal logic. Then, people learn specific instances of and definite usages of the symbol system as well as what is derived from those instances and forms of cognitively based understanding of the internal logic of mathematical symbolic systems.

The cognitive aspects needed in this framework are primarily decision processes by which members of cultural groups either accept or reject an ethnomodel as part of their own repertoire of mathematical knowledge. We believe that the conjunction of these two scenarios appear to be adequate to the depth needed to encompass the full range of cultural mathematical phenomena. In effect, there are two ways in which we recognize, represent, and make sense of a mathematical phenomenon that impinges upon our sensory apparatus:

a) First, there is a level of cognition that we share, to varying degrees, with members of our own and other cultural groups. This level would include cognitive models that we may elaborate on at a non-conscious level, which serves to provide an internal organization of external mathematical phenomena to provide the basis upon which a mathematical practice takes place.

b) Second, there is a culturally constructed representation of external mathematical phenomena that also provides its internal organization. However, this representation arises through the formulation of abstract and conceptual structures that provide forms and organizations for external phenomena.

In other words, cultural constructs provide representations for systems taken from reality. The implications for mathematical modelling are that models of cultural constructs are considered as symbolic systems organized by the internal logic of members of cultural groups. We agree with Eglash *et al.* (2006) and Rosa and Orey (2010b) who argued that models built without a first-hand sense for the world being modeled should be viewed with suspicion.

Investigators and educators, if not blinded by their prior theory and ideology, should come out with an informed sense of distinctions that make a difference from the point of view of the mathematical knowledge of the work being modeled. In so doing, they should, in the end, be able to tell outsiders (etic) what matters to insiders (emic).

Ethnomodelling respects the organization and presentation of mathematical ideas and procedures developed by the members of distinct cultural groups by constructing ethnomodels of practices found in sociocultural systems (ROSA; OREY, 2010a), which link cultural heritage with the development of mathematical practice. It is our understanding that this approach may help the organization of pedagogical action that occurs in classrooms using the emic and etic aspects of mathematical knowledge found in the school community.

Ethnomodels

Culture is a lens that shapes reality, as well as it is a blueprint that specifies a plan of action or expectations. At the same time, there are aspects of a culture that are unique to the members of distinct cultural groups, who together have grown, learned, and act daily in diverse contexts, such as economic, social, cultural, political, and environmental in which they live (ROSA; OREY, 2017).

According to Rosa and Orey (2010a), research in ethnomodelling is linked to mathematical practices developed by members of different cultural groups that tend to be of benefit to the presentation and organization of mathematical ideas and procedures that enable the development of communication, diffusion, and further transmission through generations (emic approach).

The representative idea of this approach gives room for the development of mathematical knowledge through scientific methods that may help researchers and educators to build and understand the world (etic approach) by using small units of information called *ethnomodels* that make up the set of these representations (ROSA; OREY, 2010b).

In this regard, ethnomodels are considered cultural constructs because one of the main objectives of its elaboration is to comprehend the way of thinking of these members, as well as to understand how they organize and model their mathematical ideas and procedures from their own point of view in order to mathematize their own reality (ROSA; OREY, 2012). On the other hand, a model built without a first-hand sense for the world being modeled should be viewed with suspicion.

Researchers and educators, if not hindered by their prior ideology, paradigms, cosmologies, and worldviews, should come out with an informed sense of the distinctions that are effective from the point of view of the phenomena being modeled. In so doing, they should be able to inform the outsiders (etic/global) what matters to the insiders (emic/local) (ROSA; OREY, 2017).

Ethnomodelling emphasizes the organization and presentation of mathematical ideas and procedures developed by the members of distinct cultural groups to facilitate its communication and transmission across generations, which adds cultural aspects to the modelling process. In this regard, these members construct ethnomodels of their mathematical practices found in their sociocultural systems, which link their cultural heritage to the development of the greater mathematical practices (ROSA; OREY, 2017).

This approach helps the organization of pedagogical action in classrooms by using emic (local) and etic (global) aspects of mathematical knowledge through the development and elaboration of ethnomodels, which are described as cultural artifacts that are the pedagogical tools used to enable the understanding of systems taken from the reality of the members of distinct cultural groups (ROSA; OREY, 2016).

In this regard, ethnomodels may be considered as external representations that are precise and consistent with the 2014scientific and mathematical knowledge that is socially constructed, developed, and shared by members of specific cultural groups. One objective for the elaboration of ethnomodels is to translate emic constructs which highlight the unique mathematical ideas, procedures, and practices in order to establish relations between local conceptual knowledge and the mathematics embedded in these constructs through dialogical relations (EGLASH *et al.*, 2006).

Thus, ethnomodels help to link the development of mathematical practices to the cultural heritage of the members of distinct cultural groups, who detain necessary information to solve problems and situations described in systems taken from their own reality (ROSA; OREY, 2016). The emphasis of ethnomodelling research considers the processes that help the construction and development of local mathematical knowledge systems, which include collectivity, creativity, and inventively (ASCHER, 2002) through the elaboration of ethnomodels.

According to this approach, it is impossible to imprison mathematical ideas, procedures, and practices in registers of univocal designation of reality because there are distinct systems that provide unambiguous representations of reality as well as universal explanations (CRAIG, 1998). This means that mathematics cannot necessarily be conceived as a universal language because its principles are not always the same everywhere around the world (ROSA; OREY, 2007).

Or at very least, it serves like different dialects or accents in the greater mathematical language of humanity. In accordance with this context, the production process of mathematical ideas, procedures, and practices operates within the register of interpretative singularities regarding possibilities for symbolic construction of local mathematical knowledge (ROSA; OREY, 2012).

Ethnomodelling studies local mathematical processes developed by the members of distinct sociocultural groups. Many interesting ethnomodels have been formulated by using data obtained from studies related to ethnomathematics, and which propose a rediscovery of knowledge systems adopted by the members of diverse groups (BASSANEZI, 2002; ROSA; OREY, 2012). When this knowledge applies mathematical ideas and procedures through the elaboration of ethnomodels, we can understand the origin of mathematical practices more efficiently.

In ethnomodelling research, emic (local) constructs represent the accounts, descriptions, and analyses of mathematical ideas, procedures, and practices expressed in terms of conceptual schemes and categories that are regarded as meaningful and appropriate by members of the cultural group under study. This means that emic constructs are in accordance with the perceptions and understandings deemed appropriate by the insider's culture. The validation of these constructs comes with a matter of consensus from those who do the mathematics under study, in which local people who must agree that emic constructs match shared perceptions, behaviors, and knowledge that portray characteristics of their culture (LETT, 1996).

Emic (local) mathematical knowledge can be obtained through elicitation and observation because observers infer local perceptions. In emic (local) approaches, researchers and educators must put aside their own bias, prior theories, and assumptions to let those who do the activity under study to explain, and allow for understanding mathematical themes, patterns, and concepts that emerge locally. Some of its strength lies in the appreciation of the uniqueness of the context being studied in its respect for local viewpoints, and its potential to uncover unexpected mathematical findings (ROSA; OREY, 2012).

For example, Lett (1996) states that etic constructs are accounts, descriptions, and analyses of mathematical ideas, procedures, and practices expressed in terms of conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers. An etic approach uses these concepts as starting point theories, hypothesis, perspectives, and concepts from outside of the cultural setting being studied, which are developed by researchers and educators.

Etic constructs are precise, logical, comprehensive, replicable, and observer-researcher independent (ROSA; OREY, 2017). The validation of etic approaches becomes a matter of logical and empirical analysis, in particular, the logical analysis of whether the construct meets the standards of comprehensiveness and logical consistency of concepts (LETT, 1996).

It is important to emphasize that the particular research technique used in the acquisition of scientific and mathematical knowledge has little to no bearing on the nature of that knowledge. Etic (global) approaches may be obtained at times through elicitation, as well as observation. One of the strengths of the etic (global) approach is that it allows for comparison across contexts and populations, and the development of more general cross-cultural concepts (ROSA; OREY, 2010a).

On the other hand, Rosa and Orey (2012) state that etic ethnomodels are mathematical representations elaborated through a descriptive and external observations. Thus, the representation of local mathematical knowledge is developed when the members of different cultural groups have their own interpretation of their culture (emic approach) as opposed to the researchers' interpretation and researchers who develop representations of mathematical knowledge place from the perspective of their own conceptions (etic approach).

THE DIALOGICAL APPROACH INTO A MATHEMATICS CURRICULUM

Mathematical knowledge of the members of cultural groups combined with Western-mathematical knowledge systems may result in a dialogical approach to mathematics education. An emic analysis of a mathematical phenomenon is based on internal structural or functional elements of a particular cultural group while an etic analysis is based on predetermined general concepts external to that cultural group (LOVELACE, 1984).

The emic (local) approach provides both internal conceptions and perceptions of mathematical ideas and concepts while the etic approach provides the framework for determining the effects of those beliefs on the development of the mathematical knowledge. In this perspective, the acquisition of mathematical knowledge is based on the applications of current mathematics curriculum, which may be assessed based on multiple instructional methodologies across various cultures.

In this regard, it could indeed be that one of the reasons for failure in many educational systems is that curriculum developers by using a *one size fits all* program, have ignored unique emic (local) approach in the school cultures. A dialogical approach includes the recognition of other epistemologies, and of holistic and integrated natures of mathematical knowledge of members of the diverse cultural groups found in many schools and urban centers. In other words, an ethnomodelling curriculum provides an ideological basis for learning with and from the diverse cultural and linguistic elements presented by members of distinct cultural groups (ROSA; OREY, 2010a).

In this kind of curriculum, it would be crucial to understand that an etic construct is a mathematical-theoretical idea that is assumed to apply in all cultural groups while an emic construct is one that applies only to members of specific cultural groups. This means that there is concern for cultural bias occurring if educators and researchers assume that an emic construct is etic (EGLASH *et al.*, 2006), which results in an imposed etic approach in which a culture-specific idea is wrongly imposed on the members of another cultural group.

An ethnomodelling curriculum that combines key elements of local and academic approach in a dialogical approach is likely to produce students who can manage knowledge and information systems taken from their own reality and creatively apply this knowledge to other situations. This means that ethnomodelling can be considered part of a critical mathematics education because it provides a learning process in which teachers encourage a critical examination of multiple sources of knowledge and theories found in diverse learning styles.

In this approach, acquired knowledge is centered, located, oriented, and grounded on the cultural background/context of the students, which could be applied and translated appropriately by them and thus equip them to be fully productive locally and globally. According to Rosa and Orey (2010b), ethnomodelling is a pedagogical approach to reach this goal.

The nature of the previous mathematical knowledge of the students lends themselves to the principle of sequencing in curriculum development. By giving educators the freedom to start with previous mathematical knowledge and experience of their students, we can move from the familiar to the unfamiliar and from the concrete to the abstract in the process of promoting the acquisition of mathematical knowledge (ROSA; OREY, 2006). As well we can move from emic approach to etic approach and vice versa.

In a dialogical context such as outlined here, an ethnomodelling curriculum provides the underlying philosophy for knowledge generation and exchange within and between all subsystems of mathematics education. Key elements of an ethnomodelling curriculum approach ensure the balanced integration of the affective domain of educational objectives that are essential to the recognition and utilization of the students' previous knowledge.

FINAL CONSIDERATIONS

As we discussed in this article, it is important to highlight that an emic (local) observation of mathematical practices sought to understand them from the perspective of the internal dynamics and relationships as influenced within the culture of its members. On the other hand, an etic approach provides a cross-cultural contrast and comparative perspectives by using some aspects of academic mathematics to translate this phenomenon in order to amplify the understanding of those from a different cultural background. This approach is necessary to comprehend and explain this mathematical practice as a whole from the point of view of that from the outside.

In this context, the emic (local) approach clarifies the intrinsic cultural distinctions of mathematical procedures and techniques while the etic (global) approach seeks objectivity as an outside observer across cultures regarding the development of mathematical practices. This is the dialogical approach, which concerns the stability of relationships between two distinct cultural approaches. It is important to state here that both perspectives are essential to understanding and comprehending human behaviors (PIKE, 1954), which help members of distinct cultural groups to shape mathematical ideas, procedures, and practices they developed overtime through history.

As well, it is necessary to state that one of the latest trends in mathematics education points out to the need to integrate the teaching of this science with other knowledge areas in an interdisciplinary fashion at all levels of education. For this process to be successful as well as mathematics to be valued as a discipline whose contents can be considered as a human creation, it is necessary to understand and modify the environment we live. In this regard, we can use ethnomodelling to link theory into practice by the inclusion of the dialogical approach into the mathematics curriculum. Where other than academic forms of mathematics are valued, protected, archived and most importantly shared equally.

Defined in this manner, the usefulness of the emic (local) and etic (global) distinction seems to be evident. For example, like all human beings, researchers, and educators, have been enculturated to some particular cultural worldview. They therefore need a means of distinguishing between the answers they derive as enculturated members of distinct cultural groups and the answers they derive as observers. Defining emics (local) and etics (global) in epistemological terms provides a reliable means of making that distinction.

In this perspective, culture is a lens that shapes our reality and it can be considered as a blueprint that specifies a plan of action. At the same time, a culture is unique to a specific group of people. Thus, by conducting investigations provided by both approaches: emic (local) and etic (global), we acquire a more complete understanding of the culture of the members of distinct cultural groups.

According to the discussion provides n this article, we have offered an alternative goal for the conductions of research in mathematics education, which is the acquisition of both *emic* (global) and *etic* (global) approaches for the implementation of ethnomodelling. Emic (local) knowledge is essential for an intuitive and empathic understanding of mathematical ideas of a culture, and it is essential for conducting effective ethnographic fieldwork.

On the other hand, etic (global) approach is essential for cross-cultural comparison, the essential components of ethnology, because such comparison necessarily demands standard units and categories. We also offered here a third approach on ethnomodelling research, which is the dialogical approach that makes use of both emic and etic approach and understandings through the processes of dialogue.

Finally, we define ethnomodelling as the study of mathematical phenomena within a culture because it is a social construction, it is culturally bound, and it adds the culturas features of mathematics into the mathematical modelling process.

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