



PLANTINGA AND THE BAYESIAN JUSTIFICATION OF BELIEFS

PLANTINGA E A JUSTIFICAÇÃO BAYESIANA DE CRENÇAS

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To Agnello, my father.

ABSTRACT – This article intends to show that Plantinga's criticism against Bayesianism as an account of what is involved in rationality does not apply to all forms of Bayesianism. Swinburne's version, based on a logical theory of probability, is an example of Bayesianism not hit by Plantinga's criticism. In addition, the article argues that the problem of dwindling probabilities – pointed out by Plantinga in *Warranted Christian Belief* (2000) – vanishes in a Bayesian approach. So, even if it is not a sufficient account of rationality, Bayesianism helps to understand important elements of inductive reasoning, especially those relative to cumulative cases.

KEYWORDS – Plantinga. Bayesianism. Induction. Cumulative case. Dwindling probability. Swinburne.

RESUMO – o artigo pretende mostrar que a crítica que Alvin Plantinga faz contra o bayesianismo como descrição do que está envolvido na noção de racionalidade não se aplica a toda forma de bayesianismo. A abordagem de Swinburne, baseada em uma teoria lógica da probabilidade, é um exemplo de bayesianismo não atingido pela crítica de Plantinga. Além disso, o artigo defende que, em uma abordagem bayesiana, desaparece o problema da probabilidade decrescente, apontado por Plantinga em *Warranted Christian Belief* (2000). Assim, mesmo que não seja uma descrição suficiente da noção de racionalidade, o bayesianismo ajuda a entender importantes elementos presentes no raciocínio indutivo, especialmente os relativos aos argumentos cumulativos.

PALAVRAS-CHAVE – Plantinga, Bayesianismo. Indução. Argumento cumulativo. Probabilidade decrescente. Swinburne.

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1 Plantinga and the Flaws of Subjective Bayesianism

As preliminary groundwork before putting forward his own epistemological proposal in *Warrant and Proper Function* (1993b), Alvin Plantinga criticizes in *Warrant: the Current Debate* (1993a) all major attempts at describing the element that is added to true justified belief so that it can be considered knowledge. Among the dismissed attempts is Bayesianism, which contends that for a belief to be rational it must comply with two major constraints: coherence and conditionalization. Plantinga argues that these conditions, if taken in their full sense, are either insufficient or contrary to what one would expect from a rational human being's doxastic behavior (Plantinga, 1993a, p. 115). In this text, I do not intend to go into much detail about Plantinga's analysis of the Bayesian approach, but just as much as is needed to discuss, in Bayesian terms, his dismissal, in *Warranted Christian Belief* (2000), of a cumulative evidential justification of the Christian faith.

Plantinga construes Bayesianism as a theory of belief that interprets it in terms of degree, measured according to probabilities. The probabilities involved in the Bayesian interpretation are normative or "epistemic", Plantinga says, instead of factual or statistical, since they measure the degree to which a belief is made probable in view of a set of relevant beliefs to which it is related. The usual Bayesian manner of ascribing a degree to a belief is by means of the betting behavior idea, that is, the amount of money the believer is prepared to bet on the proposition concerned. From the betting behavior comes the reason for the first Bayesian constraint on the rationality of belief degrees: coherence. In other words, according to mainstream Bayesians (the so-called "subjectivists"), coherence is a requirement for a belief to be considered rational because otherwise the believer would be subjected to a Dutch book – a situation in which no matter what happens he will always lose money in his bets. Irrationality here would basically then mean a behavior which results in imposing upon oneself net financial loss, which is an unacceptable attitude from a cost-benefit or utilitarian point of view.

The second constraint upon a belief degree, such that it can be evaluated as rational from a Bayesian point of view, is conditionalization or probability kinematics. According to it, my belief degrees must change in proportion to the probability of the propositions I learn. Since epistemic belief is a degree in conditional probability, and since the coming up of a proposition not taken into account yet changes the conditional probability, the believer's belief degree must be updated accordingly. Again, mainstream Bayesianism justifies the conditionalization constraint based on Dutch book arguments.

In his objections against the Bayesian condition of coherence, Plantinga points out some problems in the assumptions made by the description of belief degree by means of betting behavior: why should I both bet on A and not- A so that I can be caught by a Dutch book? Why should I bet at all in what I say I believe? Wouldn't it be much more rational to refuse to bet than to bet "coherently"? (Plantinga 1993a, p. 139).

As to his criticisms to the conditionalization principle, Plantinga remarks about what he calls the believer's first credence function or his "Ur-function": why should it have that importance? Why should I update the degree of beliefs that are not related to the one I learned new evidence about? Why can't I change only the probability of belief A on belief B ($P(A/B)$) without changing the probability of my beliefs in A and in B taken in isolation ($P(A)$ and $P(B)$, respectively)? (Plantinga 1993a, p. 147).

2 Cumulative Evidence and the Problem of Dwindling Probabilities

As was said above, this paper aims to assess Plantinga's account of Bayesianism mostly in relation to his criticism in *Warranted Christian Belief* (2000) of a historical case in favor of the Gospel's reliability as God's revelation. According to Plantinga, this defense of the rationality of Christian belief falls short of grounding the credence degree compatible with faith because of the problem of dwindling probabilities. In other words, if we use the probability calculus for assessing how much each piece of evidence increases the belief degree in the Gospel we end up seeing that the more evidence we have the lower the degree we get.

The aim of Plantinga's argument is to evaluate the epistemic probability degree of central Christian doctrines, whose conjunction he symbolizes by G . According to him, in the historical case approach the issue at stake is to assess the probability of G in relation to background propositions K , that is $P(G/K)$. The method is to find out propositions that add up to K and to weight the probability of G in view of this sequence of conjunctions, that is $P(G/T&K)$, $P(G/A&T&K)$, and so forth. In order to accomplish this, Plantinga suggests we use one of the axioms of probability calculus according to which $P(X/Y) \geq P(X/Z&Y) \times P(Z/Y)$. For many pieces of evidence in favor of G , each piece enters in the calculation as a probability to be multiplied, that is:

$$P(G/K) \geq \frac{P(E/K&T&A&B&C&D) \times P(D/K&T&A&B&C) \times P(C/K&T&A&B) \times P(B/K&T&A) \times P(A/K&T) \times P(T/K)}{P(B/K&T&A) \times P(A/K&T) \times P(T/K)}$$

in which:

T = there is a God

- A = God would make some kind of revelation to humankind
- B = Jesus teachings were such that they could be sensibly interpreted and extrapolated to G
- C = Jesus rose from the dead
- D = In raising Jesus from the dead, God endorsed his teachings
- E = The extension and extrapolation of Jesus' teachings to G is true

Plantinga attributes high values to the epistemic probability of each of these propositions. However, due to the fact that they are multiplied, the result ends up being below to 0.5, and this is too little for the conviction force normally expected in faith. According to Plantinga (2000, p. 280):

The main problem for such a historical case, as I see it, is what we can call the principle of dwindling probabilities: the fact that in giving such a historical argument, we can't simply annex the intermediate propositions to K (as I'm afraid many who employ this sort of argument actually do) but must instead *multiply* the relevant probabilities.

As mentioned above, in his analysis, Plantinga uses a formula of conditional probability according to which $P(X/Y) \geq P(X/Z \& Y) \times P(Z/Y)$, which is an expression of the third axiom of the probability calculus also known as the multiplication law. Now, the multiplication law provides the probability of an event E happening given the occurrence of other events F, G, H etc. In other words, using Plantinga's example (2000, p. 273): knowing that 0.9 is the probability that Eleonore (E) is at the party *given that Paul (P) is also there* (and everything else we know about the world (K)) – in formal terms $P(E/P \& K)$ –, and given that the probability that Paul is at the party ($P(P/K)$) is 0.9 too, what is the probability that Eleonore is at the party ($P(E/K)$)? To get the answer, we just need to apply the probability law above, and we will find $P(E/K) \geq P(E/P \& K) \times P(P/K)$, which is 0.81, a lower figure than the original ones, which is what happens when you multiply fractions. Now, why should we multiply the probabilities of Eleonore being at the party if Paul is there and the one that Paul is at party to obtain the result? Because they refer to occurrences that are supposed to happen together, and the figure is lower because if it is uncertain that X will happen, it is even more uncertain that X will happen in conjunction with Y.

However, Plantinga does not provide any reason why his formula gives a good account of epistemic probability, that is, the probability of a thesis in view of evidence. The way he formalizes the question only means that the greater the amount of happenings the less probable it is that they will occur together. Yet, this is not the way we relate a proposition

we want to test in view of a set of other propositions. In other words, the fact that a man is found stabbed to death in his flat has a low probability, the fact that there is a knife near the corpse is also improbable in itself, the presence of Jones's fingerprints on the knife may also be improbable, and the fact that Jones might have interest in the victim's death may have an intrinsic low probability too. Still, all these facts are even less probable if considered conjointly. However, a very different question is assessing how much all these evidences make probable the hypothesis that Jones has murdered the victim.

In other words, it is one thing to say that all the events $T, A \dots E$ have a low probability of happening together, but it is a very different one to evaluate the probability of G in view of those happenings. While the joint probability of G occurring together with $T, A \dots E$ is necessarily lower the more items you consider (given that their probabilities are not zero or one), the epistemic probability of G in view of those propositions may either increase or decrease, depending on how much the total evidence available confirms G .

So, as a first argument against Plantinga's account, it seems intuitively weird that the more evidence you have in favor of an idea the less this idea gets confirmed. In addition, if we use the Bayesian method, the strange result Plantinga found does not obtain, and perhaps this is a good reason to think that Bayesianism is not so useless as an interpretation of rationality as Plantinga argued.

3 Many Types of Bayesianism and Cumulative Probability

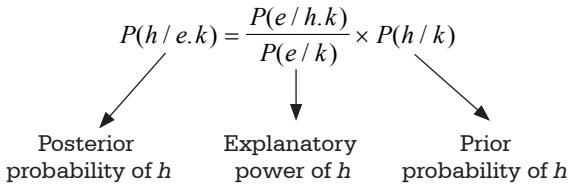
As says the title of a paper by the statistician Irving Good, there are (at least?) 46.656 varieties of Bayesians (Good, 1983). So, for example, Richard Swinburne's Bayesianism is not the same as the one Plantinga expounded and criticized in *Warrant: the Current Debate* (1993a).

All Bayesians assume that belief is something that can be measured by probability. One of the crucial differences among them is the theory of probability they assume. Subjective probability, stated in terms of betting behavior, is the most common amongst them, but it is not the only one. Swinburne, for example, assumes the so-called logical theory of probability, according to which probability "is a measure of the extent to which one proposition r makes another one q likely to be true (r and q may be complicated conjunctions or disjunctions of other propositions)" (Swinburne, 2001, p. 62). This type of probability is concerned with the extent to which a proposition provides reason for believing another one.

It is from the logical relationship that characterizes inductive probability that Swinburne extracts the concept of logical probability,

which will be crucial to his probabilistic theory. Logical probability is a type of inductive probability in which the inductive support that a proposition q gives to a proposition r is measured not only by all the relevant logical possibilities and corresponding entailments, but also by the correct inductive criteria. A value that ideally could only be reached by a logically omniscient being, but “[...] to which we try to conform our judgments of inductive probability on evidence but about the value of which we may make mistakes”, says Swinburne (2001, p. 64). In other words, the value of a logical probability is totally determined a priori, according to the logical relationship between the actual contents of the propositions concerned and the correct inductive criteria known by a logically omniscient being.

As said above, another important common trace among Bayesians is the idea that the assessment of a hypothetical belief h (that is its ‘posterior probability’) should be done with the help of Bayes’s theorem, which is deduced from the axioms of probability calculus. So, Bayes’s theorem permits to distinguish between the prior probability of h and its explanatory power, formalized as follows (in the notation below, ‘.’ means ‘and’. ‘ e ’ stands for the evidence and ‘ k ’ for the background knowledge apart from e):

$$P(h / e.k) = \frac{P(e / h.k)}{P(e / k)} \times P(h / k)$$


While in the subjective theory prior probability is the personal degree of belief by a subject S , in the logical theory it is the degree of belief permitted by correct criteria of induction, such as the principle of simplicity (the simpler a theory the more probable it is *ceteris paribus*), for instance. The coherence constraint is also assumed in the logical theory, but it is taken as measure of logical consistency. As to conditionalization, this is viewed as the way we should update our prior probability in view of evidence, so that if the posterior probability is higher than the prior, the hypothesis has been confirmed by evidence; yet, if the posterior is lower than the prior, the hypothesis has been disconfirmed by evidence.

According to the conditionalization rule the prior probability of h at any one point is a function of its posterior probability in the preceding link in the chain. Applying this notion, we have the following means of

calculating the probability of a hypothesis in the light of many cumulative pieces of evidence in Bayes's theorem:

- for evidence e_1 :
$$P(h/e_1,k) = \frac{P(e_1/h,k) \times P(h/k)}{P(e_1/k)}$$
- for evidence e_2 :
$$P(h/e_2,e_1,k) = \frac{P(e_2/h,k,e_1) \times P(h/e_1,k)}{P(e_2/k,e_1)}$$
- for evidence e_3 :
$$P(h/e_3,e_2,e_1,k) = \frac{P(e_3/h,k,e_1,e_2) \times P(h/e_2,e_1,k)}{P(e_3/k,e_1,e_2)}$$
- for e_7 :

$$P(h/e_7,e_6,e_5,e_4,e_3,e_2,e_1,k) = \frac{P(e_7/h,k,e_1,e_2,e_3,e_4,e_5,e_6) \times P(h/e_6,e_5,e_4,e_3,e_2,e_1,k)}{P(e_7/k,e_1,e_2,e_3,e_4,e_5,e_6)}$$

This is the notion presupposed in Swinburne's inductive cumulative case for theism. Separately, each piece of evidence in its favor may be weak, but when taken jointly, they can make up a considerably strong argument (Swinburne, 2004). However, Swinburne's insistence on the principle of simplicity sometimes obscures the crucial importance of conditionalization in a Bayesian analysis. So, what will be said below is not intended as a defense of Swinburne's approach, but of what I see as a better interpretation of the Bayesian method applied to the problem of dwindling probabilities.

Now, applying the above ideas to the problem put forward by Plantinga, a Bayesian analysis needs to ascribe figures to two concepts that Plantinga has not employed: the prior probability of the thesis under assessment and the probability of each piece of evidence given background knowledge. Before going into it, let us say a brief word about using numbers in this kind of reasoning. It certainly sounds artificial stating that the prior belief degree in the Christian Gospel has a definite numerical value for someone, but this artifice is very useful to see what follows from our beliefs, which is an important means to evaluate their reasonableness. In addition, the assignment of a value is a very common practice in academic evaluation, for example, where a student's performance in a test or assignment is ordinarily described in numeric terms.

Bearing this in mind, from a Bayesian point of view, the first estimate lacking in Plantinga's analysis is the prior probability of G (belief in the

Christian Gospel). Let us assume a very low degree, typical of someone who does not believe it. In addition, following Plantinga (see 2000, p. 274ff.), let us assign high probabilities to pieces of evidence from T to E in view of G , i.e. $P(T \dots E/G)$. Apart from this, we need to attribute a value to each piece of evidence in view only of natural background knowledge (i.e. excluding G). Given the religious character of the evidence enlisted, it seems reasonable to attribute a low epistemic probability for T (the first piece considered) given the other beliefs the skeptic has, let us put it at 0.2 then. Bear in mind that the prior probability of each piece of evidence will have to cohere with previous evidence accrued to the background knowledge, so that the starting figure may increase or decrease depending on what means the evidence added up, that is, on whether they confirm or not the hypothesis.

Now, following Plantinga's instances of evidence, let us assess the probability of $P(G/T \dots E \& K)$, taking each piece in turn.

Recalling the considerations above, we may plausibly consider that for the skeptic who ascribes to G (the Gospel) a prior probability of 0.1, the probability of T (theism) in itself is also low, let us say 0.2, but that, in view of the truth of G (the central Christian doctrines) ($P(T/G \& K)$), T gets very probable. Again, following Plantinga's assignment, let us ascribe 0.9 to it.

In this case, $P(G/T \& K)$ will be

$$P(G/T \& K) = \frac{P(T/G \& K)}{P(T/K)} \times P(G/K)$$

Using the figures indicated above, we will have $P(G/T \& K) = 0.45$. In other words, T (the existence of God), even if not very probable in itself for the skeptic, is able to raise G 's probability from 0.1 to 0.45 if calculated in the Bayesian way.

The next piece of evidence ($A =$ God would make some kind of revelation to humankind) will be considered in Bayes's theorem as follows:

$$P(G/T \& A \& K) = \frac{P(A/G \& K \& T)}{P(A/T \& K)} \times P(G/T \& K)$$

Now the prior probability of G is the posterior probability obtained in the preceding calculation (0.45). Let us assume again 0.9 for the likelihood of A given G , K and T , and ascribe a value to $P(A/T \& K)$ that is plausible for a rational skeptic: 0.5 let us say. A "rational skeptic" in a Bayesian sense is someone who is both coherent with his religious skepticism and

with the axioms of the probability calculus, so that he will not assign to $P(A/T\&K)$ a value that will render a posterior probability of G that is higher than 1 or lower than 0. The result will be 0.81, which is another step in the confirmation of G .

One more exercise of the type above will be enough to ground some of the concluding considerations of this article. Let us take into account the following piece of evidence provided by Plantinga: $B =$ Jesus teachings were such that they could be sensibly interpreted and extrapolated to G . In the theorem, B will be introduced this way:

$$P(G/T\&A\&B\&K) = \frac{P(B/G\&T\&A\&K)}{P(B/T\&A\&K)} \times P(G/T\&A\&K)$$

Similar considerations will be made here as regards the figures involved. The prior probability of G is the result of the preceding calculation (0.81). The likelihood of B (its probability given $G\&T\&A\&K$) will again be 0.9, and the expectancy of B (its probability given all evidence already known in the process – except G , which is not a piece of evidence, but the hypothesis under assessment) will need to rise accordingly. Let us assume 0.8 for our reluctant but rational skeptic. The result will be 0.91 this time.

I will save the reader's time and forgo further numbers and calculations. The morals of this story is that, in opposition to Plantinga's principle of dwindling probability, the more evidence considered in favor of the Christian doctrines, the more it gets confirmed by it, as long as the Bayesian method of calculating the probabilities concerned is assumed. However, although a hypothesis may get increasingly confirmed by favorable pieces of evidence, the constraints of the probability calculus will see to it that it will never reach the value of 1, which figure corresponds to absolute certainty. As John Earman said in another context, there is a "curious blend of inductivism and anti-inductivism that flows from the Bayesian analysis" (Earman 2000, p. 28). In other words, the Bayesian analysis is able to provide a rigorous account of inductive reasoning in contexts of uncertainty. However, it also shows that no matter how many pieces of evidence are given in favor of a hypothesis, it will never get totally confirmed; there will always be some room for doubt.

4 Concluding Remarks

So, if Plantinga were to use the Bayesian method, the probability of the Gospel's reliability as God's revelation would not diminish at the

consideration of each favorable piece of evidence, but rather *increase*, up to a level very close to 1, which is a degree equivalent to a very strong belief.

Now, strong belief is not the same as certainty, which is presupposed in the attitude of faith. So, perhaps Plantinga is right in saying that only reason is not enough to ground faith and something else, like the instigation of the Holy Spirit for example is also needed to take a human being to faith, even if the problem of dwindling probabilities is resolved.

It is not the place to go deeper into this question. Yet, it might be interesting to notice that an alternative to Plantinga's stance is Swinburne's position that belief is just a component of faith, the theoretical one. The practical element of faith, which justifies it can be praised, since it depends on voluntary decision, is commitment. Conviction in the Christian view is perhaps better interpreted as a practical commitment instead of theoretical certainty. If so, the Christian faith is compatible with a degree of belief not equal to 1 in probabilistic terms. The ideal faithful may have some small space for doubt, but his practice is entirely devoted to God's will. Swinburne elaborates this point in *Faith and Reason* (2005), and I comment on it and compare his approach to Plantinga's in Portugal (2011).

From what has been said so far, the Bayesian concept of rationality includes three elements. Firstly, it postulates that in order to be rational one must be logically consistent, which is the coherence constraint put in terms of the logical theory of probability. Secondly, it demands that our belief degree should be updated in light of new information, which corresponds to an interesting dynamic dimension of rationality. Thirdly, in order that a belief degree may be considered rational, it must be proportional to the degree of confirmation provided by the total evidence available to the believer. Apart from this, Bayes's theorem may be taken as an useful instrument for assessing our belief degrees and the way it changes in view of all we know, and as such an important algorithm able to evaluate the rationality of our beliefs and our inductive inferences in the sense above.

In addition, Swinburne's Bayesian approach exemplifies the fact that rebutting subjective Bayesianism does not mean rebutting Bayesianism itself. Plantinga is probably right in saying that Bayesians do not give a sufficient account of either warrant or rationality. Even so, one still might say that they give interesting clues to what is involved in inductive reasoning from a formal point of view, and this matters quite a bit to those interested in the assessment of beliefs.

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Recebido em 08/03/2012.

Aprovado para publicação em 09/07/2012.