Relevance and goal conciliation: logical adequacy and empirical plausibility

Relevância e conciliação de metas: adequação lógica e plausibilidade empírica

Suelen Francez Machado Luciano¹, Fábio José Rauen²

ABSTRACT: Wilson (2004) designs an example to explain the notion of cognitive effects employing three assumptions as initial cognitive context: (1) “I’ll (probably) catch the bus” (2) “If I catch the bus, I’ll get to the lecture” and (3) “If I don’t catch the bus, I’ll miss the lecture” and two possible achievements: case A, the individual catches the bus and gets to the lecture, and case B, the individual does not catch the bus and misses the lecture.

In the example, Wilson (2004) has to preview at least two opposite options in the initial cognitive context to use only modus ponens as an elimination rule, suggesting that this arrangement is necessary in all uses of conditionals. In this article we argue that the biconditional architecture can be applied to the example, decreasing the initial processing cost and keeping the logical adequacy. We also argue that a goal conciliation architecture can explain not only cases where options like yes/no can be anticipated, but also situations where options like yes/no-alternatives are viable, increasing the empirical plausibility of the analysis.

KEYWORDS: Cognitive pragmatics; Relevance theory; Goal conciliation theory; Logical adequacy; Empirical plausibility.

RESUMO: Wilson (2004) concebe um exemplo para explicar a noção de efeitos cognitivos empregando três suposições como contexto cognitivo inicial: (1) “Eu (provavelmente) pegarei o ônibus”, (2) “Se eu pegar o ônibus, eu irei à conferência”, e (3) “Se eu não pegar o ônibus, eu perderei a conferência”; e duas possibilidades de realização: caso A, no qual o indivíduo pega o ônibus e vai à conferência, e caso B, no qual o indivíduo não pega o ônibus e perde a conferência. No exemplo, Wilson (2004) tem de prever pelo menos duas alternativas opostas de consecução no contexto cognitivo inicial para utilizar como regra dedutiva de eliminação unicamente a regra de modus ponens, sugerindo que esta providência é necessária em todos os usos de condicionais. Nós argumentamos neste artigo que a arquitetura bicondicional pode ser aplicada ao exemplo, diminuindo o custo de processamento inicial e mantendo a adequação lógica. Nós também argumentamos que a arquitetura de conciliação de metas pode explicar não apenas situações nas quais opções de tipo sim/não podem ser antecipadas, mas também situações nas quais opções de tipo sim/não alternativas são viáveis, aumentando a plausibilidade empírica da análise.

PALavrAS-CHave: Pragmática cognitiva; Teoria da relevância; Teoria de conciliação de metas; Adequação lógica; Plausibilidade empírica.
**Introduction**

Wilson (2004, lesson 3, p. 3-5) shows the cognitive effects of processing an input in context – strengthening of a contextual assumption; contradicting and eliminating a contextual assumption; and combining with a contextual assumption to yield a contextual implication – with an example in which someone intends to catch a bus to attend a lecture.

She suggests the following context in the case.

Anna is running for her bus in the morning, with the following thoughts in her mind (which constitutes the context in which the new information will be processed):

1. I’ll (probably) catch the bus.
2. If I catch the bus, I’ll get to the lecture.
3. If I don’t catch the bus, I’ll miss the lecture.

This initial cognitive context can be formalized with three probable assumptions.

1. \( P \) probable initial cognitive context
2. \( P \rightarrow Q \) probable initial cognitive context
3. \( \neg P \rightarrow \neg Q \) probable initial cognitive context

Next, Wilson presents two versions of the example: case A, where the individual catches the bus; and case B, where the individual does not catch the bus.

In the first case, she adds the following information.

(4a) The bus is coming towards.

According to Sperber and Wilson (1995), such information is processed by a deductive module, which analyzes a set of assumptions and deduces all the possible conclusions of this set, working with elimination rules like elimination-e, modus ponens and modus tollens in a non-trivial and non-demonstrative way. This device is “an automaton with a memory and the ability to read, write and erase logical forms, compare their formal properties, store them in memory and access the deductive rules contained in the logical entries for concepts” (p. 94-95). In short, the device strengthens assumptions which are confirmed; it contradicts and eliminates assumptions which are not confirmed; and it combines new or newly presented assumptions with stored assumptions, yielding contextual implications. Thus, information on the bus coming towards (4a), in the set of the contextual assumptions (1-3), strengthens the first assumption (5a) and yields the contextual implication (6a).

(5a) \( P \) I will catch the bus (strengthening of (1)).
(6a) \( Q \) I will get to the lecture (by modus ponens of (2) and (5a)).

In the second case, Wilson (2004) adds the following information.

(4b) The bus has pulled away from the stop.

In the set of the contextual assumptions (1-3), information (4b) contradicts and eliminates assumption (1), yielding assumption (5b), and combines with assumption (3), yielding the contextual implication (6b).

(5b) \( \neg P \) I will not catch the bus (contradiction and elimination of (1)).
(6b) \( \neg Q \) I will not get to the lecture (by modus ponens of (3) and (5b)).
In both cases, only two out of the three assumptions of the initial cognitive context are processed. The device processes only the first and the second assumptions in case A and processes only the first and the third assumptions in case B, i.e., it works nicely only where modus ponens is applicable. Although Wilson does not say anything about the residual assumptions, they are supposedly erased.4

Wilson’s (2004) example shows the cognitive effects of processing an input in a context correctly, but it has the contradictory feature of increasing the initial processing cost of the deductive device with two opposite assumptions (2-3). Wilson needs to deal with them in the initial cognitive context in order to preserve the precedence of modus ponens in such inferences.5 So, our first task in this study is to propose an alternative way to describe and explain the example, without overload the deductive device, keeping logical accuracy and exemplifying all the cognitive effects showed by Wilson.

Extending the scope of Wilson’s (2004) example to empirical domains, we can think about its plausibility. The requirement of opposite assumptions in every conditional situation seems counterintuitive. If the process is always dichotomous, as Wilson’s (2004) modeling suggests, then human beings live in a constant state of doubt or alert. We do not deny there are “yes/no” or “yes/no-alternative” cases in advance. However, there are a lot of situations in which people even conceive failures. In such cases, people think about failures or even think about alternatives only when they deal with problems.

Furthermore, we need to call into question what determines the initial filtering of assumptions P and Q, and the interpretative process itself. Since Wilson (2004) uses a material conditional, “getting to the lecture” Q is conditioned by “catching the bus” P. However, it seems that the process is geared by the goal of “getting to the lecture” rather than geared by the evidence of “catching the bus”.

Rauen (2014) argues that the relevance-theoretic deductive machinery is part of a process which starts with a goal designing. In his goal conciliation theory, he states that the individual (a) fixes the effect Q, (b) searches the best ante-factual abductive hypothesis with the lowest processing cost to achieve the effect Q, and (c) verifies conciliations of the achievements Q’ with the original goal Q. For him, the goal Q plays a crucial role to explain both, the initial filtering of the premises P and the emergence of alternative solutions in cases of non-conciliations. Considering those intuitions, we will verify if such an alternative architecture can describe and explain how Anna deals with situations in which the bus has just pulled away from the stop, in order to increase the empirical plausibility of the modeling.

Finally, we will analyze a verbal version of the example to verify whether the abductive-deductive modeling is applicable in communicative interactions. For that, we take the case in which Anna runs to the bus stop with the same thoughts (1-3) in her mind, and sees Beatriz, who says that the bus is coming (4c) or has passed (4d):

(4c) Beatriz: The bus is coming.
(4d) Beatriz: The bus has passed.

The utterance (4c), similar to the perception of the bus coming towards (4a), makes Anna conclude that she will get to the lecture; and the utterance (4d), similar to the perception of the bus pulling away from the

4 Luciano (2014) pursues a line of argument according to which the assumptions (2) and (3) can be processed independently. In such cases, the deductive device is faced with fallacies when the assumption (5a) is processed as antecedent of the second conditional (assumption (3)), and the assumption (5b) is processed as antecedent of the first conditional (assumption (2)). According to her, Wilson’s (2004) example works nicely only if we consider together the two opposite assumptions, but such a conclusion is an inference that is not explicitly expressed in the initial cognitive context.

5 The argument here is that the initial cognitive context of the deductive device must always have two opposite assumptions if modus ponens is the only rule to be used in conditional contexts.
stop (4b), makes Anna conclude that she will miss the lecture. So, the same questions addressed to the original non-verbal cases can be addressed to verbal ones.

1 Logical adequacy

We explore in this section deductive alternatives to describe and explain Wilson’s (2004) example, avoiding the overload of the initial cognitive context of the deductive device. For that, we will consider the following elimination rules: modus ponendo ponens, modus ponendo tollens with a conjunction of alternatives, modus ponendo tollens with an exclusive disjunction of alternatives, and biconditional.

Our first alternative is to use modus ponendo ponens only with the first and the second assumptions of Wilson’s (2004) example.

(1) \( P \) I’ll (probably) catch the bus.
(2) \( P \rightarrow Q \) If I catch the bus, I’ll get to the lecture.

Now, information on the bus coming towards (3a), in the set of the contextual assumptions (1-2), strengthens the first assumption (4a) – Anna will catch the bus – and yields the contextual implication (5a) – Anna will get to the lecture.

(3a) The bus is coming towards.
(4a) \( P \) I will catch the bus (strengthening of (1)).
(5a) \( Q \) I will get to the lecture (by modus ponens of (2) and (4a)).

On the other hand, information on the bus pulling away from the stop (3b) contradicts and eliminates assumption (1), yielding assumption (4b) – Anna will not catch the bus. However, assumption (4b) in the context of the conditional assumption (2) yields a fallacy of denying the antecedent, resulting in both getting to the lecture \(*Q\) (5b’) or not \(*\neg Q\) (5b”).

(3b) The bus is pulling away from the stop.
(4b) \( \neg P \) I will not catch the bus (contradiction and elimination of (1)).
(5b’) \(*Q\) I will get to the lecture (by denying the antecedent of (2) and (4b)).
(5b”) \(*\neg Q\) I will not get to the lecture (by denying the antecedent of (2) and (4b)).

Our first alternative reduces the overload of the initial cognitive context, but it has the inconvenience of yielding fallacy when the bus pulls away from the stop. By the way, this is one of the reasons why Wilson has supposedly designed two opposite assumptions in her example. In addition, the elimination of the third assumption also does not express the state of doubt suggested in Wilson’s (2004) example.

The second alternative is using modus ponendo tollens with a conjunction of opposite alternatives, launching both conditionals proposed by Wilson’s (2004) example in the assumption (2) of the initial cognitive context.

(1) \( P \) I’ll (probably) catch the bus.
(2) \( \neg((P \rightarrow Q) \land \neg(P \rightarrow \neg Q)) \) It is not true that simultaneously if I catch the bus, I’ll get to the lecture; and if I don’t catch the bus, I’ll miss the lecture.

Let us observe the effect of such an option in case A. Seeing the bus coming towards (3a) strengthens the first assumption (4a) – Anna will catch the bus. Next, assumption (4a) strengthens the first conditional of the conjunction (5a), contradicting and eliminating the second one, and yields the assumption (6a) by modus ponens – Anna will get to the lecture.

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6 We use asterisks to highlight that there is a logical problem in the description.
(3a) The bus is coming towards.

(4a) \( P \) I will catch the bus (strengthening of (1)).

(5a) \( P \rightarrow Q \) If I catch the bus, I’ll get to the lecture

(strengthening of the first conjunct conditional of (2)).

(6a) \( Q \) I will get to the lecture (by modus ponens of (4a) and (5a)).

On the other hand, seeing the bus pulling away from the stop (3b) in case B, contradicts and eliminates the first assumption (4b) – Anna will not catch the bus. Next, assumption (4b) strengthens the second conditional of the conjunction (5b), contradicting and eliminating the first one, and yields the assumption (6b) by modus ponens – Anna will not get to the lecture.

(3b) The bus is pulling away from the stop.

(4b) \( \neg P \) I will not catch the bus (contradiction and elimination of (1)).

(5b) \( \neg P \rightarrow \neg Q \) If I don’t catch the bus, I’ll miss the lecture

(strengthening of the second conjunct conditional of (2)).

(6b) \( \neg Q \) I will not get to the lecture (by modus ponens of (4b) and (5b)).

So, modus ponendo tollens with a conjunction of opposite options decreases the quantity of propositions in the initial cognitive context, avoiding the fallacy of denying the antecedent of our first alternative (modus ponendo ponens without Wilson’s (2004) original third assumption). Moreover, this alternative also implies the possibility that both alternatives can be false simultaneously, as in cases where someone waits the bus without knowing if it has passed or not. However, despite the correctness of the logical procedure, it is hard to conceive people actually think that way: modus ponendo tollens with a conjunction of opposite options, grounded on denying a conjunction of alternatives, is not psychologically realistic, because it demands a very complex propositional calculus.

Furthermore, such a path goes against the intuition that people tend to model options by exclusive disjunctions like “or this, or that.” We can use modus tollendo ponens with an inclusive disjunction \( P \lor Q \) or with an exclusive disjunction \( P \lor Q \). Inclusive disjunction is not adequate here, because it models situations where the two disjunctive assumptions can be true simultaneously. Exclusive disjunction, in turn, is logically acceptable, but it has the undesirable effect of taking the perception of the bus coming towards as the denial of its departure to imply, in the next step, the conclusion that someone can take the bus – and vice versa. In short, what is odd when this rule is applied is taking an assumption as a denying, and a denying as a denying of one affirmation; when it is much more reasonable (and relevant) to deal with affirmation or denial directly.

One non-orthodox way to deal with this oddity is using modus ponendo tollens in such cases. According to Heimbeck (1969), it is important to stress that whether the modus ponendo tollens is always fallacious in inclusive disjunctions, it can be considered valid logically in exclusive disjunctions of opposite assumptions. So, our third alternative is to use modus ponendo tollens in an exclusive disjunction context.

(1) \( P \) I’ll (probably) catch the bus.

(2) \( (P \rightarrow Q) \lor (\neg P \rightarrow \neg Q) \) Or if I catch the bus, I’ll get to the lecture;

or if I don’t catch the bus, I’ll miss the lecture.

Let us observe the effect of such a procedure in case A. Seeing the bus coming towards (3a) strengthens the first assumption (4a) – Anna will catch the bus. Next, assumption (4a) strengthens the first conditional of the exclusive disjunction (5a), contradicting and eliminating the second one, and yields the assumption (6a) by modus ponens – Anna will get to the lecture.
(3a) The bus is coming towards.

(4a) \( P \) I will catch the bus (strengthening of (1)).

(5a) \( P \rightarrow Q \) If I catch the bus, I'll get to the lecture
(strengthening of the first exclusive disjunct conditional of (2)).

(6a) \( Q \) I will get to the lecture (by modus ponens of (4a) and (5a)).

On the other hand, seeing the bus pulling away from the stop (3b) in case B, contradicts and eliminates the first assumption (4b) – Anna will not catch the bus. Next, assumption (4b) strengthens the second conditional of the exclusive disjunction (5b), contradicting and eliminating the first one, and yields assumption (6b) by modus ponens – Anna will not get to the lecture.

(3b) The bus is pulling away from the stop.

(4b) \( \neg P \) I will not catch the bus (contradiction and elimination of (1)).

(5b) \( \neg P \rightarrow \neg Q \) If I don't catch the bus, I'll miss the lecture
(strengthening of the second exclusive disjunct conditional of (2)).

(6b) \( \neg Q \) I will not get to the lecture (by modus ponens of (4b) and (5b)).

Although modus ponendo tollens applied to exclusive disjunctive assumptions is an elegant solution – it shows the state of doubt expressed in Wilson's (2004) example and decreases the quantity of assumptions in the initial cognitive context –, the second assumption of the architecture seems to demand the same processing cost required by two separate opposite assumptions. So, we explore the possibility of a biconditional modeling to describe and explain the case with lower initial cognitive cost.

In Wilson's (2004) example, a biconditional architecture has the advantage of compiling assumptions (2) “If I catch the bus, I'll get to the lecture” \( P \rightarrow Q \) and (3) “I don't catch the bus, I'll miss the lecture” \( \neg P \rightarrow \neg Q \) of the original Wilson's (2004) example with a single command \( P \leftrightarrow Q \) "If and only if I catch the bus, I'll get to the lecture." So, our fourth alternative is to use a biconditional formulation as the second assumption in the initial cognitive context of the deductive device.

(1) \( P \) I'll (probably) catch the bus.

(2) \( P \leftrightarrow Q \) If and only if I catch the bus, I'll get to the lecture.

Let us observe the effect of such an option in case A. Seeing the bus coming towards (3a) strengthens the first assumption (4a) – Anna will catch the bus. Next, assumption (4a) eliminates the biconditional (5a) and yields assumption (6a) by modus ponens – Anna will get to the lecture.

(3a) The bus is coming towards.

(4a) \( P \) I will catch the bus (strengthening of (1)).

(5a) \( P \leftrightarrow Q \) If I catch the bus, I'll get to the lecture
(by elimination of the biconditional).

(6a) \( Q \) I will get to the lecture (by modus ponens of (4a) and (5a)).

On the other hand, seeing the bus pulling away from the stop (3b) in case B, contradicts and eliminates the first assumption (4b) – Anna will not catch the bus. Next, assumption (4b) eliminates the biconditional (5b) and yields assumption (6b) by modus ponens – Anna will not get to the lecture.

(3b) The bus is pulling away from the stop.

(4b) \( \neg P \) I will not catch the bus (contradiction and elimination of (1)).

(5b) \( \neg P \rightarrow \neg Q \) If I don't catch the bus, I'll miss the lecture
(by elimination of the biconditional).

(6b) \( \neg Q \) I will not get to the lecture (by modus ponens of (4b) and (5b)).
Thus, the biconditional architecture decreases the initial processing cost of the deductive device – there is no need to consider two opposite assumptions to model the state of doubt in Wilson’s (2004) example. So, it is a better way to describe and explain “yes/no” cases, in spite of assuming the denying of the antecedent when the bus pulls away from the stop, amplifying the set of logical rules traditionally used in relevance-theoretic literature.

If in Wilson’s (2004) description two opposite situations in every “yes/no” conditional must be designed in the initial cognitive context, we can infer that every alternative must be designed in advance in “yes/no-alternative” cases to bear the deductive architecture. This sounds counterintuitive, especially because people tend to think about alternatives only when faced with problems, i.e., they realize problems first and think about solutions next. Our intuition is that only an abductive/deductive modeling can describe and explain such cases. So, we will explore Rauen’s (2014) goal conciliation theory to model this extended version of Wilson’s (2004) example.

2 Goal conciliation theory

Rauen’s (2014) goal conciliation theory aims to connect the notions of relevance and goal. He argues that the expansion of the cognitive context is crucially abductive, and people are moved by intended conclusions rather than by premises. In his theory, “the agent abducts a hypothesis or inference to the best solution, principle of plausibility, which simultaneously is the solution with the lowest cost faced with the fixed effect of the goal, principle of relevance” (2014, p. 613, italics in the original).

Goal conciliation architecture has four stages: goal designing, and ante-factual abductive hypothesis formulation, execution, and checking.

In the fourth stage, the author designs four types of achievements using the theoretical notion of goal conciliation: active conciliation, active non-conciliation, passive conciliation, and passive non-conciliation; and five architectures for ante-factual abductive hypotheses evaluation: categorical, biconditional, conditional, enabling, and tautological.

The first stage of Rauen’s architecture is goal designing, that is formalized as follows (2014, p. 599):

\[1\] The individual \( i \) designs a goal \( Q \) at the time \( t_1 \).

Such that:

a) The time \( t_1 \) represents the instance of the goal designing; and
b) The goal \( Q \) is a future state that does not exist at the time \( t_1 \).

In Wilson’s (2004) example, goal designing is the time \( t_1 \) in which Anna \( i \) decides to get to the lecture \( Q \).

\[1\] Anna \( i \) designs the goal \( Q \) of getting to the lecture at the time \( t_1 \).

Or, schematically:

\[1\] \( Q \) get to the lecture, Anna

The second stage of Rauen’s architecture is the formulation of an ante-factual abductive hypothesis, that is formalized as follows (2014, p. 599-600):

\[2\] The individual \( i \) abducts an ante-factual hypothesis \( H_a \) to achieve the goal \( Q \) at the time \( t_2 \).

Such that:

a) The time \( t_2 \) is the instance of the formulation of the ante-factual abductive hypothesis \( H_a \).
b) The time $t_2$ succeeds the time $t_1$;

c) The ante-factual abductive hypothesis $H_a$ corresponds to a formulation like “If $P$, then $Q$” so that $P$ is an antecedent action and $Q$ is a consequent state;

d) The goal $Q$ is admitted by the individual $i$ as a consequent state in the scope of the ante-factual abductive hypothesis $H_a$;

e) The antecedent action $P$ is admitted by the individual $i$ as at least probably sufficient to achieve the consequent state $Q$ in the scope of the ante-factual abductive hypothesis $H_a$;

f) The ante-factual abductive hypothesis $H_a$ is the first formulation which is consistent with the principle of relevance, because it has the lowest processing cost faced with the fixed effect projected by the consequent state $Q$;

g) Simultaneously, the ante-factual abductive hypothesis $H_a$ is taken by the individual $i$ as an inference to the best plausible solution to achieve the consequent state $Q$.

The outcome can be formalized as follows in Wilson’s (2004) example:

$$[2a] \text{Anna } i \text{ abducts the best ante-factual abductive hypothesis } H_a \text{ to achieve the goal } Q \text{ of getting to the lecture at the time } t_2.$$  

The output of formulation $[2a]$ needs to be completed, because it does not identify the antecedent action $P$ that is admitted as at least enough to achieve the consequent state $Q$. Then, let us consider that the following factual assumptions $S_{1-3}$ are in Anna’s encyclopedic memory:

$S_1$ – Bus takes to the lecture.

$S_2$ – Taxi takes to the lecture.

$S_3$ – Ride takes to the lecture.

In such a restricted context, all the assumptions $S_{1-3}$ are plausible, because Anna can get to the lecture by bus, taxi, or ride. Based on this set of factual assumptions, it is plausible to admit that the most relevant and feasible solution is getting to the lecture by bus $S_1$.

According to Rauen (2014, p. 600), there are four criteria to an assumption to be assumed as the best hypothesis: it must be mapped by a hypothetical formulation; it must contain an action at least enough to achieve the goal; it must be the hypothesis with the lowest processing cost; and it must be considered as the best solution. So, the more relevant and plausible ante-factual abductive hypothesis $H_a$ in that case is:

$$[2b] \text{Anna } i \text{ abducts that if Anna catches the bus, then Anna will get to the lecture at the time } t_2.$$  

Or, schematically:

$$[1] \begin{array}{c} Q \text{ get to the lecture, Anna} \\ [2] P Q \text{ catch the bus, Anna get to the lecture, Anna} \end{array}$$

The third stage in Rauen’s architecture (2014, p. 601-602) is the execution of the antecedent action. According to the author, the individual adopts the positive model by default, i.e., he/she performs the antecedent action.

$$[3a] \text{the individual } i \text{ performs } P \text{ to achieve } Q \text{ at the time } t_3; \text{ or}$$

$$[3b] \text{the individual } i \text{ does not perform } P \text{ to achieve } Q \text{ at the time } t_3.$$

Such that:

a) The time $t_3$ is the instance of the execution of the antecedent action $P$ in the context of the hypothetical formulation “If $P$, then $Q$”;

b) The time $t_3$ succeeds the time $t_2$;

c) The model $[3b]$ is implied by the inaction in $[3a]$;

d) The inaction can be voluntary or involuntary.$^9$

$^8$ Assuming arbitrarily that taxi is most expensive and rides are less likely to be achieved.

$^9$ The negative model can occur in voluntary or involuntary situations. For example, Anna could not have caught the bus because she came to the bus stop a few moments later on purpose, or because the bus has passed before the schedule.
The active output of the third stage can be as follows in Wilson’s (2004) example:

[3a] Anna i catches the bus to get to the lecture at the time $t_3$.

Or, schematically:

\[ [1] \quad Q \quad Q \quad \text{get to the lecture, Anna} \]
\[ [2] \quad P \quad Q \quad \text{catch the bus, Anna} \]
\[ [3] \quad P \quad \text{catch the bus, Anna} \]

The fourth stage is checking the ante-factual abductive hypothesis. At this stage, according to Rauen (2014, p. 603), the individual evaluates the antecedent action $P$, checking the hypothetical formulation “If $P$, then $Q$” deductively.\(^{10}\)

\[ [4a] \quad \text{the individual } i, \text{ considering } [2] \text{ “If } P \text{ then } Q' \text{” and } [3a] \text{ “} P \text{” achieves } Q' \text{ at the time } t_4; \text{ or} \]
\[ [4b] \quad \text{the individual } i, \text{ considering } [2] \text{ “If } P \text{ then } Q' \text{” and } [3b] \text{ “} \neg P \text{”, achieves } \neg Q' \text{ at the time } t_4. \]

Such that:

a) The time $t_4$ is the instance of achieving the goal $Q$;
b) The time $t_4$ succeeds the time $t_3$;
c) The model [4a] is the model of the achievement of the action $P$ [3a], and the model [4b] is the model of the achievement of the inaction $\neg P$ [3b];
d) The consequent state $Q'$ is the outcome of the action $P$ [3a], and the consequent state $\neg Q'$ is the outcome of the inaction $\neg P$ [3b];
e) The consequent state $Q'$ or $\neg Q'$ is an actuality at the time $t_4$.\(^{11}\)

\(^{10}\)The fourth stage converges with the relevance-theoretic deductive module.

\(^{11}\)The expression $Q'$ highlights that the external achievement of the goal is different in some way from its internal projection. Semantically, in more complete or complex descriptions, the very instances of $Q$ could be indexed by numbers $Q_1, Q_2, Q_3, \ldots, Q_n$ so that $Q_i$ stands for the instance of the goal design. Syntaxically, the distinction between $Q$ and $Q'$ is irrelevant, because all the instances of $Q$ are exemplars of the consequent state $Q$. The same reasoning should be applied to the antecedent action $P$. On the differences between internal and external goals, see Tomasello et al. (2005).
The four possibilities for goal achievements can be seen in the following table:

Table 1 – Possibilities for goal achievements

<table>
<thead>
<tr>
<th>Stages</th>
<th>Active conciliation (a)</th>
<th>Active non-conciliation (b)</th>
<th>Passive conciliation (c)</th>
<th>Passive non-conciliation (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
</tr>
<tr>
<td>[2]</td>
<td>P</td>
<td>Q</td>
<td>P</td>
<td>Q</td>
</tr>
<tr>
<td>[4]</td>
<td>Q’</td>
<td>~Q’</td>
<td>Q’</td>
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Next, Rauen (2014, p. 604) defines the notion of confirmation of an ante-factual abductive hypothesis $H_a$, i.e., “the state $Q'$ at the time $t_4$ [that] satisfies, coincides, or corresponds with the hypothesis $H_a$ at the time $t_2$.” He argues that the evaluation of an ante-factual abductive hypothesis $H_a$ “depends on the degree of confidence or strength which is attributed to the hypothesis.” Based on this idea, he classifies such hypotheses into categorical, biconditional, conditional, enabling and tautological ones.

A **categorical ante-factual abductive hypothesis** is a formulation $P\Rightarrow Q$, whose truth table returns “true” only when $P$ and $Q$ are true. In such a case, $P$ and $Q$ are sufficient, necessary, and certain; and the only achievement admitted by the individual is the **active conciliation (a)**.

[...]

In a **biconditional ante-factual abductive hypothesis $P\Leftrightarrow Q$**, are true the cases where both $P$ and $Q$ are true or false. Categorical abductive hypotheses become biconditional in inactions $\neg P$, whether in involuntary problems or in voluntary dilemmas. In such cases, the individual admits **passive non-conciliations (d)**, and the mere consideration of the option $\neg P\Rightarrow \neg Q$ weakens the first categorical hypothetical formulation. $P$ and $Q$ are now sufficient and necessary, but not certain.

In a **conditional ante-factual abductive hypothesis $P\rightarrow Q$**, the antecedent action becomes a sufficient but not necessary condition for the subsequent state $Q$, so that the material implication applies. In this case, there is a further weakening of the strength of the abductive hypothesis because the individual also admits passive conciliations (c).

In an **enabling ante-factual abductive hypothesis $P\leftarrow Q$**, the antecedent action $P$ becomes necessary, but not sufficient to achieve the consequent state $Q$. The action $P$ enables, but does not guarantee the consequent state $Q$. This allows admitting active non-conciliations (b).

Finally, in a **tautological ante-factual abductive hypothesis $P\equiv Q$**, $P$ and $Q$ are not sufficient, necessary, or certain, modeling situations like “If $P$, then possibly $Q$” In such a case all types of achievements are possible. (RAUEN, 2014, p. 605-606).

The following truth table summarizes such ideas.

Table 2 – Truth table for modeling ante-factual abductive hypotheses

<table>
<thead>
<tr>
<th>Conciliations</th>
<th>Terms</th>
<th>Categorical</th>
<th>Biconditional</th>
<th>Conditional</th>
<th>Enabling</th>
<th>Tautological</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Active Conciliation</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(b) Active Non-Conciliation</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(c) Passive Conciliation</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>(d) Passive Non-Conciliation</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>


Rauen states that ante-factual abductive hypotheses emerge as categorical $P\Rightarrow Q$ by definition and lose strength insofar the obstacles appear. He argues that the individual is usually moved by the conviction that the goal will be achieved. In Wilson’s (2004) example, Anna believes that catching the bus is an enough, necessary and certain action to get to the
lecture. Faced with an obstacle, the hypothesis is weakened to a bi-conditional $P \leftrightarrow Q$ level, because passive non-conciliations come into consideration. So, the action $P$ is enough and necessary to achieve the goal $Q$, but is no more certain. Wilson’s (2004) example is modeled at this level, as we show in the second section, because only if Anna catches the bus, then she gets to the lecture.

There are also cases where the hypothesis is said to be conditional $P \rightarrow Q$, i.e., when the antecedent action is enough, but not necessary for the consequent state $Q$. In such cases, passive conciliations come into consideration. In Wilson’s (2004) example, that can happen when someone offers a ride suddenly.

Another option is considering that catching the bus does not imply getting to the lecture. Here, the hypothesis is enabling $P \leftarrow Q$, in a way that the antecedent action $P$ is now necessary, but not enough to achieve the consequent state $Q$. In such cases, passive non-conciliations come into play. In Wilson’s (2004) example, that can happen when someone catches the bus and does not arrive on time to attend the lecture.

Finally, weakening even more the connection between the antecedent action $P$ and the consequent state $Q$, the ante-factual abductive hypothesis can be tautological $P \rightarrow Q$. In a tautological architecture, the antecedent action $P$ is not enough nor necessary to achieve the consequent state $Q$, modeling situations type “If $P$, then possibly $Q$.” In such cases, all the achievements are possible.

3 Empirical plausibility in Wilson’s (2004) example

Let us see how the notions of conciliation and confirmation are applicable in Wilson’s (2004) example, assuming that the ante-factual abductive hypothesis is categorical. In an active conciliation (a), Anna catches the bus $P$ and the achievement $Q'$ is conciliated with the goal $Q$ – Anna gets to the lecture. Hence, the hypothesis $H_a$ “If I catch the bus, I’ll get to the lecture” is strengthened and stored in the encyclopedic memory as a factual assumption to be triggered in similar scenarios. The more conciliation, the less will be the processing cost to reuse it as the first abductive hypothesis in similar contexts.

Each stage of the active conciliation (a) can be seen below:

1. $Q$ Anna projects getting to the lecture.
2. $P \leftrightarrow Q$ Certainly, if Anna catches the bus, then Anna gets to the lecture.
3. $P$ Anna catches the bus.
4. $Q'$ Anna gets to the lecture.

In a passive non-conciliation (d), Anna does not perform the action of catching the bus, and the achievement $\neg Q'$ does not conciliate with the goal $Q$ of getting the lecture accordingly. The hypothesis $H_a$ is confirmed and stored as a factual assumption to be triggered in future situations. In such a case, there are at least two cognitive effects: the weakening of the categorical hypothesis to a biconditional $P \leftrightarrow Q$ level [4], and the implied conclusion [5] that Anna will not get to the lecture $\neg Q$.

1. $Q$ Anna projects getting to the lecture.
2. $P \leftrightarrow Q$ Certainly, if Anna catches the bus, then Anna gets to the lecture.
3. $\neg P$ Anna does not catch the bus.$^{13}$
4. $P \leftrightarrow Q'$ If, and only if Anna catches the bus, she gets to the lecture.
5. $\neg Q$ Anna will not get to the lecture.$^{13}$

$^{12}$Wilson’s (2004) example models the certainty of catching the bus in the case A, but not the certainty of getting to the lecture. Therefore, scenarios of goal conciliations are only hypothetical projections.

$^{13}$From now on the asterisks represent both, voluntary and involuntary inactions, and breaks like fails, frustrations, accidents, surprises, etc.
It is worth mention that if we stop at step [5], the outcome of Rauen’s architecture is similar to Wilson’s description – Anna gives up the goal. However, we argue that such a situation cannot be the end of the processing, especially if Anna really wants to get to the lecture. So, Anna must weigh the strength of the assumptions $Q$ and $\neg Q'$. If $\neg Q'$ prevails, then Anna truly gives up; but if $Q$ prevails, a new problem arises – Anna must abduct a new ante-factual hypothesis to get to the lecture, and calling a taxi, for instance, can be considered in our restricted set of assumptions $S_{1-3}$.

In an active non-conciliation (b), Anna catches the bus, but the achievement $\neg Q'$ does not conciliate with the goal $Q$ – Anna does not get to the lecture. Then, the strength of the hypothesis $H_a$ is weakened to an enabling level, once the antecedent action $P$ is not enough to achieve the consequent state $Q$. Here, the lack of conciliation is relevant, because there is now a dilemma between giving up and persist. In such a case, Anna must weigh the strength of the assumptions $Q$ and $\neg Q'$, as mentioned in passive non-conciliation.

In a passive conciliation (c), Anna does not catch the bus, but the reality $Q'$ is conciliated with the goal $Q$ of getting to the lecture (as in a sudden ride, for example). The sudden achievement of goal $Q$ is not enough to reject the hypothesis $H_a$ that the bus allows getting to lectures, although there may be some effect of weakening. In such a case, the hypothesis is weakened to a conditional level, once the bus is enough, but not necessary to get to the lecture.

Until now, we have shown processing possibilities when an ante-factual abductive hypothesis emerges as categorical. But there are cases in which the hypothesis can emerge in any of the other possibilities. In a biconditional context, the individual considers “yes/no” or “all/nothing” situations. Wilson’s (2004) example is configured in such a way, because the fact that Anna is running enables us to infer that something made her think about missing the lecture.

This type of situation can be described as follows.

So, Anna’s case can be modeled in one out of the four possibilities bellow:
Table 3 – Conciliations of a biconditional ante-factual abductive hypothesis

<table>
<thead>
<tr>
<th>Stages</th>
<th>Active conciliation (a)</th>
<th>Active non-conciliation (b)</th>
<th>Passive conciliation (c)</th>
<th>Passive non-conciliation (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
</tr>
<tr>
<td>[2]</td>
<td>P⁻⁺Q</td>
<td>P⁻⁺Q</td>
<td>P⁻⁺Q</td>
<td>P⁻⁺Q</td>
</tr>
<tr>
<td>[4]</td>
<td>Q’</td>
<td>~Q’</td>
<td>~Q’</td>
<td>~Q’</td>
</tr>
<tr>
<td>[5]</td>
<td>P⁻⁻Q</td>
<td>P⁻⁻Q</td>
<td>Q⁻⁻Q’</td>
<td>Q⁻⁻Q’</td>
</tr>
<tr>
<td>[6]</td>
<td>Q⁻⁻Q’</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
</tr>
<tr>
<td>[7]</td>
<td>Q</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


In the case of an **active conciliation** (a), Anna catches the bus and gets to the lecture, and in the case of a **passive non-conciliation** (d), Anna does not catch the bus and does not get to the lecture, as previewed by the biconditional hypothesis. In the first case, the abductive/deductive modeling does not diverge of a purely deductive modeling. In the second case, and differently of an exclusively deductive modeling, Anna can weigh the strength of the achievement ~Q against the strength of the goal Q, and insist in achieving the goal (calling a taxi, for instance). In the case of an **active non-conciliation** (b), the hypothesis is weakened to an enabling level, because Anna catches the bus, but something happens, making her think that she cannot get to the lecture. Once more, Anna can weigh the strength of the achievement ~Q against the strength of the goal Q, and insist in achieving the goal (going on foot, for instance). Finally, in the case of a **passive conciliation** (c), the hypothesis is weakened to a conditional level (someone offers a ride suddenly). The bus is now enough, but not necessary to get to the lecture.

The achievements above have modeled what Rauen (2014) calls self-conciliated scenarios. In such scenarios, the individual himself checks the achievements in the scope of an ante-factual abductive hypothesis. However, there are cases that require conciliation of interests between more than one individual. So, we will explore Rauen’s (2014) notion of goal hetero-conciliation, extending Wilson’s (2004) example even more, to consider two utterances: case A, in which Beatriz says that the bus is coming; and case B, in which Beatriz says that the bus has passed.

In our first verbal version of the example, Beatriz says that the bus is coming. If we are dealing with scenarios modeled by proactive goals, then Beatriz must have a goal herself. Let us take as certain that Beatriz and Anna intend to get to the lecture together and Beatriz realizes that Anna is running because she is late. So, the best ante-factual abductive hypothesis to hurry her is conveying information that the bus is coming. In such a situation, Anna and Beatriz share the goal Q of getting to the lecture together and the ante-factual abductive hypothesis P of catching the bus. What Beatriz knows, but Anna just supposes, is that catching the bus is still viable. In relevance-theoretic terms, Beatriz must convey such an information, producing an ostensive stimulus like “The bus is coming,” attracting Anna’s attention and becoming mutually manifest Beatriz’s informative intention.

Beatriz’s context can be described as follows:

1. \( Q \) Anna and Beatriz project to get to the lecture together.
2. \( P \Leftrightarrow Q \) Certainly, if Anna and Beatriz catch the bus, then they will get to the lecture together.
3. \( M \Leftrightarrow P \) Certainly, if Beatriz warns Anna that the bus is coming, then Anna and Beatriz will catch the bus together.
4. \( M \) Beatriz warns Anna that the bus is coming.
5. \( P' \) Anna and Beatriz catch the bus.
6. \( Q' \) Anna and Beatriz get to the lecture.
Anna’s initial cognitive context is slightly different, because her ante-factual abductive hypothesis to get to the lecture with Beatriz is bi-conditional. Remembering, she is not entirely confident that she can catch the bus in time.

[1] $Q$ Anna and Beatriz project to get to the lecture together.
[2] $P \leftrightarrow Q$ If and only if Anna catch the bus, then Anna and Beatriz will get to the lecture together.

So, Anna uses the relevance-theoretic comprehension procedure to process Beatriz’s utterance. She follows a path of least effort in computing cognitive effects, considering interpretations in order of accessibility, and stopping when her expectation of relevance is satisfied. In such a procedure, she fits Beatriz’s utterance into a logical form, develops an explicature, including the higher speech act, and yields implied conclusions (implicatures) when necessary.

The utterance on the bus arrival can be explicated as follows:15

Linguistic Form: The bus is coming.
Logical Form: (be coming $x$, $\alpha_{\text{place}}$, $\beta_{\text{time}}$).
Explicature: BEATRIZ WARNS ANNA THAT THE BUS IS COMING TO THE BUS STOP AT THE TIME OF THE UTTERANCE.
Expanded explicature: BEATRIZ WARNS ANNA THAT THE BUS IS COMING TO THE BUS STOP AT THE TIME OF THE UTTERANCE.

Here, the explicature is not Beatriz’s communicative intention. To achieve such an intention, Anna needs to embed the explicature in her own chain of goals.

$S_1$ – Anna and Beatriz wants to get to the lecture together (implied premise from shared goal).

$S_2$ – If and only if Anna catches the bus, then Anna and Beatriz will get to the lecture together (implied premise from Anna’s ante-factual abductive hypothesis).
$S_3$ – Beatriz warns Anna that the bus is coming to the bus stop at the time of the utterance (implied premise from the explicature of Beatriz’s utterance).
$S_4$ – $S_4 \rightarrow S_5$ (by modus ponens).
$S_5$ – Anna can catch the bus (implied conclusion).

Just after Beatriz’s utterance, supposedly, the best ante-factual abductive hypothesis to catch the bus is Anna to run faster.

Let us see the description:

[1] $Q$ Anna and Beatriz project to get to the lecture together.
[2] $P \leftrightarrow Q$ If and only if Anna catch the bus, then Anna and Beatriz will get to the lecture together.
[3] $M \leftrightarrow P$ Certainly, if Anna runs faster, then Anna will catch the bus.
[5] $P'$ Anna and Beatriz catch the bus.
[6] $Q'$ Anna and Beatriz get to the lecture.

Now let us take the case B. Information about missing the bus has two effects here: dissuade Anna from the need to run until the bus stop and, hypothetically, coordinate alternative actions that result from such a fact.

Let us see the description:

Linguistic Form: The bus has passed.
Logical Form: (pass $x$, $\alpha_{\text{place}}$, $\beta_{\text{time}}$).
Explicature: THE BUS HAS PASSED TO THE BUS STOP IN A TIME BEFORE THE UTTERANCE.
Expanded explicature: BEATRIZ WARNS ANNA THAT THE BUS HAS PASSED TO THE BUS STOP IN A TIME BEFORE THE UTTERANCE.

15 On the description procedures, see for example Rauen (2011).
The implications are the following:

- $S_1$ – Anna and Beatriz wants to get to the lecture together (implied premise from shared goal).

- $S_2$ – If and only if Anna catches the bus, then Anna and Beatriz will get to the lecture together (implied premise from Anna's ante-factual abductive hypothesis).

- $S_3$ – Beatriz warns Anna that the bus has passed to the bus stop in a time before the utterance (implied premise from the explicature of Beatriz’s utterance).

- $S_4$ – $S_3 \rightarrow S_5$ (by modus ponens).

- $S_5$ – Anna will not catch the bus (implied conclusion/implied premise).

- $S_6$ – $S_5 \rightarrow S_7$ (by modus ponens).

- $S_7$ – Anna and Beatriz will not get to the lecture (implied conclusion).

Now, as explained in self-non-conciliation scenarios, Anna and Beatriz need to solve together the situation, i.e., giving up or persist in getting to the lecture. If the strength of the goal $Q$ overcomes the achievement $\neg Q$, a new abductive cycle starts and new communicative interactions will be produced to achieve the goal. For instance, Anna and Beatriz can share the expenses of a taxi or think about how to take a ride. Anyway, they need to coordinate new goals and sub-goals collaboratively.

**Conclusion**

Wilson (2004) exemplifies relevance-theoretic cognitive effects – strengthening of a contextual assumption, contradicting and eliminating of a contextual assumption, and combining with a contextual assumption to yield a contextual implication – with an example in which someone wants to get to a lecture by bus. Wilson designs three assumptions as initial cognitive context, two of them expressing opposite conditionals. The first conditional is used in case of catching the bus (case A); and the second conditional is used in case of missing the bus (case B). With such a procedure, Wilson presents all the cognitive effects using only *modus ponens* as deductive rule.

If Wilson’s solution is correct, all the instances of conditionals imply considering at least two possibilities of achievements in advance – one modeling success and the other modeling failure – with the undesirable feature of increasing the initial processing cost of the deductive device. So, we explore deductive alternatives to describe and explain the example by keeping logical accuracy and exemplifying all the cognitive effects showed by Wilson without overloading the deductive device.

The first solution was to keep up the *modus ponens* architecture and adopt only the second assumption – in which the individual gets to the lecture catching the bus. Although this solution results in a better way to describe the success (case A), it implies accepting the fallacy of denying the antecedent in the negative version (case B), in addition to losing the state of doubt presented in Wilson’s (2004) example. Another solution was to transform the second proposition in a complex conjunction – it is not true that someone simultaneously catches the bus and gets to the lecture and does not catch the bus and misses the lecture – and to apply *modus ponendo tollens* architecture. This modeling avoided the fallacy of denying the antecedent and reduced the quantity of assumptions. However, it is hard to accept people use such a complex rule when they dispose disjunctions for that. So, we used *modus ponendo tollens* with exclusive disjunction of opposite assumptions – or someone catches the bus and gets to the lecture, or someone does not catch the bus and misses the lecture. The disjunctive modeling of opposite assumptions is more similar to the reasoning we believe people have in such situations, but it requires the same processing
cost demanded by two separate opposite assumptions. Ergo, we modeled the example with the biconditional – if and only if someone catches the bus, then someone gets to the lecture. In a biconditional architecture, catching the bus \( P \) implies getting to the lecture \( Q \), and missing the bus \( \neg P \) implies missing the lecture \( \neg Q \). So, this rule i) represents a stronger restriction that eliminates the fallacy of denying the antecedent, ii) has a lower processing cost, and iii) has the apparent advantage of modeling the failure in background.

Extending Wilson’s (2004) example to empirical plausibility concerns, all the deductive alternatives are sterile in negative scenarios. The individual never gets to the lecture if he misses the bus. This is odd, because human beings are able to design alternative ways to achieve a goal when they are faced with problems. So, we explore Rauen’s (2014) goal conciliation theory in which the notions of relevance and goal are closely connected.

Rauen (2014) argues that the relevance-theoretic deductive module is a stage of a higher abductive/deductive process. In the higher stage, the individual designs a goal and abducts the first ante-factual hypothesis – which is both the best and the most relevant solution – to achieve it. Next, the individual performs the antecedent action and checks the outcomes.

Thus, we argue that the goal \( Q \) has an essential role in filtering actions \( P \), in a way that getting to the lecture is primary in relation to catching the bus. In Wilson’s (2004) example, Anna designs the goal \( Q \) of getting to the lecture and abducts a categorical ante-factual hypothesis \( P \Leftrightarrow Q \) of catching the bus to achieve it. Nevertheless, something happens, and such a hypothesis is weakened to a biconditional level \( P \leftrightarrow Q \). At that level, seeing the bus coming towards implies getting to the lecture and seeing the bus pulling away from the stop implies missing the lecture, as in a purely deductive modeling. However, as the goal conciliation abductive/deductive modeling starts with goal designing. Anna can weigh the strength of her initial internal goal against the external non-achievement in cases of the self-non-conciliation, and a new abductive/deductive cycle can be initiated if the strength of the goal overcomes the strength of the failure.

Next, we explored a communicative scenario where Beatriz and Anna had intended to get to the lecture, and Beatriz warns Anna that the bus is coming or that it has passed. Here, they shared not only the personal goal of getting to the lecture, but also the personal goal of doing it together. In such a hetero-conciliated scenario, Beatriz’s utterances does not only make Anna know that they can get to the lecture or not in a similar way of the perception of the bus coming towards or pulling away from the stop in a self-conciliated scenario: it also predicts the emergence of alternative ante-factual abductive hypotheses in cases where they persist in getting to the event even after they have missed the bus. Such outcomes suggest that the notion of goal conciliation can be enlarged to model communicative interactions as well as it models self-conciliated scenarios like Wilson’s (2004) original example.

**References**


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